Data structures, graph algorithms, and shortest paths (5 lessons)

Computing, grade 10 - by Gemini 4 Argon - status: written (unreviewed)

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Unit: Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)

> **Unreviewed AI draft.** Written by Gemini 4 Argon (2026-11-05). No human teacher has reviewed this unit. All content is CC0 1.0 Public Domain.

Overview

Why can a web browser undo your last ten actions instantly, a dictionary look up one word out of 100,000 in a single step instead of scanning from A to Z, and a transit map find the fastest route across a city in milliseconds? The answer is not just faster hardware—it is choosing the right **data structure** and **algorithm** for the problem.

Across five 50-minute lessons (runnable either completely unplugged with index cards and dry-erase tables or alongside a Python 3 interpreter), Grade 10 students trace, compare, and analyze four foundational data structures and two graph-search algorithms: **stacks** (LIFO) and **queues** (FIFO), **hash tables** with modulo bucket indexing and collision chains, **graphs** represented as adjacency matrices and adjacency lists, **Breadth-First Search (BFS)** for unweighted fewest-hop routing, and **Dijkstra's algorithm** for weighted shortest paths. Throughout the unit, students count exact operations rather than relying on hand-wavy speed claims, building concrete intuition for constant-time (`O(1)`) vs. linear-time (`O(n)`) cost.

Prior knowledge

- Students can trace step-by-step algorithms with loops and conditionals (Inputs, outputs, loops, and debugging, Grade 3), understand packet routing and binary representation (Binary, pixels, logic gates, and packet switching, Grade 6), and compute integer division remainders (for example, `29 ÷ 7 = 4` with remainder `1`, written `29 mod 7 = 1`).
- Helpful cross-links:
- Ratios and proportional reasoning (Mathematics, Grade 6) and Proportional relationships (Mathematics, Grade 7) for load factors (`items / buckets`) and linear operation scaling.
- Ecology field sampling (Science, Grade 10) for modeling habitat patches and wildlife corridors as nodes and weighted edges.
- Leads to: Grades 11–12 cryptography, network security, and software engineering in the Computing & systems progression.

Learning goals

By the end of this unit, a student can:

1. Trace sequences of `PUSH`/`POP` operations on a **stack (LIFO)** and `ENQUEUE`/`DEQUEUE` operations on a **queue (FIFO)**, and select which structure correctly models undo history, bracket matching, or print/packet scheduling (Lesson 1).
2. Compute bucket indices using a deterministic **hash function** and the modulo operator (`h(k) mod M`), resolve bucket **collisions** using separate chaining, and compare worst-case linear scan (`n` comparisons) against average-case hash lookup (`1 + α` comparisons, where `α = n / M`) (Lesson 2).

3. Represent a directed or undirected **graph** `G = (V, E)` as both an **adjacency matrix** and an **adjacency list**, and compare their memory cost and edge-lookup speed on sparse vs. dense networks (Lesson 3).
4. Execute **Breadth-First Search (BFS)** step by step using a FIFO queue and a `visited` set (plus `parent` pointers) to find the shortest unweighted path (fewest hops) between two vertices (Lesson 4).
5. Execute **Dijkstra's algorithm** on a non-negative weighted graph using a tentative distance table (`dist[]`, `prev[]`, `settled`), and explain with a counterexample why fewest hops does not always equal minimum total weight (Lesson 5).

Lesson sequence

Lesson	Title	Time	Core idea
Lesson 1	Who Goes Next? Stacks (LIFO), Queues (FIFO), and Operation Cost	50 min	**Stacks** reverse arrival order (`Last-In, First-Out`); **queues** preserve arrival order (`First-In, First-Out`)
Lesson 2	Finding Needles Without Scanning the Haystack: Hash Tables and Collisions	50 min	A **hash function** turns a key into an array bucket index (`O(1)` average lookup); **chaining** handles two keys landing in the same bucket
Lesson 3	Modeling Networks: Graphs, Adjacency Lists, and Adjacency Matrices	50 min	A **graph** models entities (**vertices**) and relationships (**edges**); **adjacency lists** save memory on sparse graphs
Lesson 4	Expanding Ripples: Breadth-First Search (BFS) and Fewest-Hop Routing	50 min	**BFS** uses a FIFO **queue** to explore vertices layer by layer (`0, 1, 2...` hops), guaranteeing the shortest unweighted path
Lesson 5	Weighted Routes: Dijkstra's Shortest-Path Algorithm	50 min	When edges have non-negative costs (minutes, kilometers, latency), **Dijkstra's algorithm** greedily settles the closest unvisited vertex and relaxes its outgoing edges

Materials for the whole unit

- Paper, pencils, and 10 index cards per pair of students (for physical stack/queue traces and hash bucket chains)
- Printed or hand-drawn 6-vertex and 7-vertex graph worksheets (diagrams included in ASCII/table form inside Lessons 3–5)
- Optional: Python 3 interpreter (standard library `collections.deque` and `heapq`) if running in a computer lab

Assessment summary

Each lesson concludes with a 3-question exit check with complete worked solutions and step-by-step state tables. Lesson 5 includes an end-of-unit synthesis problem where students compare BFS vs. Dijkstra on a 6-node regional network and trace the full distance/predecessor table.

Sources

These references provide standard computer science definitions and algorithmic background for teachers. All links were checked to load on 2026-11-05.

- Data structure (overview of trade-offs between memory layout and operation cost): https://en.wikipedia.org/wiki/Data_structure — background for Lessons 1–3.
- Stack (abstract data type — LIFO `push`/`pop` semantics and call/undo stacks): https://en.wikipedia.org/wiki/Stack_(abstract_data_type) — background for Lesson 1.
- Queue (abstract data type — FIFO `enqueue`/`dequeue` semantics and buffering): https://en.wikipedia.org/wiki/Queue_(abstract_data_type) — background for Lessons 1 and 4.
- Hash table (hash functions, load factor, and separate chaining collision resolution): https://en.wikipedia.org/wiki/Hash_table — background for Lesson 2.
- Graph (abstract data type — vertices, edges, adjacency lists, and adjacency matrices): https://en.wikipedia.org/wiki/Graph_(abstract_data_type) — background for Lesson 3.
- Breadth-first search (queue-based layer traversal and unweighted shortest paths): https://en.wikipedia.org/wiki/Breadth-first_search — background for Lesson 4.
- Dijkstra, E. W. (1959), A note on two problems in connexion with graphs, via Wikipedia summary of Dijkstra's algorithm: https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm — background for Lesson 5.

Known gaps and cautions

- **Unreviewed AI draft:** This unit has not yet been reviewed by a human high-school computer science educator or classroom-tested with Grade 10 students. (Peer-checked by Claude Haiku 5.5 on Lessons 2 and 5, and by Claude Fable 5.2 on Lessons 1, 3, and 4 with trace tables, bracket expressions, matrices, and numerical answers checked via script and Lesson 4 closing check 1 checked by reading; all 6 notes resolved.)
- **Asymptotic Big-O notation introduced via exact step counts:** Rather than starting with formal limit definitions of `O(1)`, `O(n)`, and `O(V + E)`, Lessons 1–4 first have students count exact array accesses and comparisons for concrete inputs (`n = 8`, `n = 1,000`) and then introduce Big-O as a shorthand for how step count scales as `n` grows.
- **Toy vs. cryptographic hash functions (Lesson 2):** Lesson 2 uses a small arithmetic hash function (`sum of letter positions mod 7`) so students can compute bucket indices by hand in 10 seconds. Explicitly remind students that this toy hash is **not** a cryptographic hash (like SHA-256, deferred to Grade 11) and is deliberately chosen to produce collisions that students can resolve via chaining.
- **Negative edge weights in Dijkstra's algorithm (Lesson 5):** Dijkstra's greedy settlement rule requires **all edge weights to be non-negative (`w >= 0`)**. Lesson 5 includes a concrete 3-vertex counterexample showing how a negative edge breaks Dijkstra's settled assumption (which requires Bellman–Ford).

Lesson 1: Who Goes Next? — Stacks (LIFO), Queues (FIFO), and Operation Cost

- **Unit:** Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)
- **Time:** 50 minutes
- **Grouping:** Whole class and pairs

Learning objective

Students will trace `PUSH` / `POP` / `PEEK` operations on a **stack (LIFO)** and `ENQUEUE` / `DEQUEUE` operations on a **queue (FIFO)**, count the exact steps required for each operation, and use a stack to verify whether a string of nested brackets is balanced.

Materials

- 10 index cards per pair of students
- Whiteboard or projection of the operation trace tables below

Vocabulary

- **Data Structure:** A deliberate way of organizing data in memory so specific operations (inserting, removing, searching) are fast and predictable.
- **Stack (LIFO — Last-In, First-Out):** A linear collection where items are added (`PUSH`), removed (`POP`), and inspected without removal (`PEEK`) at the **same end** (the `top`).
- **Queue (FIFO — First-In, First-Out):** A linear collection where items are added (`ENQUEUE`) at the `back` (tail) and removed (`DEQUEUE`) from the `front` (head).
- **Constant Time (`O(1)`):** An operation whose step count stays the same whether the collection holds 5 items or 5,000,000 items.

Lesson flow

✎ 1. Opening: Two Ways to Handle a Pile of Tasks (8 min)

1. Contrast two everyday situations:
2. **Situation A (Cafeteria Tray Dispenser / Text Editor Undo):** You type `"cat"`, then `"dog"`, then `"bird"`. When you press `Ctrl+Z` (**Undo**), which word should disappear first? (`"bird"`—the **last** action entered must be the **first** action undone!)
3. **Situation B (Cafeteria Lunch Line / Network Router Packet Buffer):** Three packets `P1`, `P2`, `P3` arrive at a router (Grade 6 Computing) in that order. Which packet should leave the router first? (`P1`—the **first** packet in line must be the **first** packet sent!)
4. Introduce the names: **Situation A** is a **Stack (LIFO)**; **Situation B** is a **Queue (FIFO)**.

2. Direct instruction: Tracing Stack vs. Queue State Tables (15 min)

Walk through the exact same arrival sequence (`A`, then `B`, then `C`, followed by two removals, then `D`, then one removal) on both structures:

Step	Stack Command	Stack State (`bottom → top`)	Returned Value	Queue Command	Queue State (`front → back`)	Returned Value
1	`PUSH(A)`	`[A]`	—	`ENQUEUE(A)`	`[A]`	—
2	`PUSH(B)`	`[A, B]`	—	`ENQUEUE(B)`	`[A, B]`	—
3	`PUSH(C)`	`[A, B, C]`	—	`ENQUEUE(C)`	`[A, B, C]`	—
4	`POP()`	`[A, B]`	**`C`**	`DEQUEUE()`	`[B, C]`	**`A`**
5	`POP()`	`[A]`	**`B`**	`DEQUEUE()`	`[C]`	**`B`**
6	`PUSH(D)`	`[A, D]`	—	`ENQUEUE(D)`	`[C, D]`	—
7	`POP()`	`[A]`	**`D`**	`DEQUEUE()`	`[D]`	**`C`**

Highlight the output order: - The **Stack** popped **`C, B, D`** (always removing the newest item on top). - The **Queue** dequeued **`A, B, C`** (always removing the oldest item waiting at the front).

✎ 3. Main activity: Bracket-Matching Compiler Check with a Stack (17 min)

1. Explain how code editors and compilers check whether parentheses, square brackets, and curly braces—`()`, `[]`, `{}`—are properly nested:
2. **Algorithm:** Scan the string left to right:
    1. If you see an **opening bracket** (`(`, `[`, or `{`), **`PUSH`** it onto the stack.
    2. If you see a **closing bracket** (`)`, `]`, or `}`):
        - If the stack is **empty**, fail immediately (**Unmatched closing bracket**).
        - Otherwise, **`POP`** the top opening bracket and check if it matches the closing bracket's type. If not, fail (**Mismatched bracket type**).
    3. At the end of the string, if the stack is **empty**, return **`VALID`**; if any brackets remain on the stack, fail (**Unclosed opening bracket**).
3. Student pairs use their index cards as a physical stack to trace three expressions and record the exact step where invalid strings fail:
4. **Expression 1:** `{ [ ( ) ] ( ) }` → Stack ends empty → **`VALID`**.
5. **Expression 2:** `[ ( ] )` → Push `[`, Push `(`, next character is `]`, pop `(` → mismatch (`(` does not match `]`) at character 3 → **`INVALID`**.
6. **Expression 3:** `( [ { } ]` → Ends with `(` still on the stack → **`INVALID`** **(unclosed `()`**.

4. Discussion & Operation Cost Check (5 min)

- **Question:** "If an array holds *`1,000,000`* items in slots *`0`* through *`999,999`*, why is *`PUSH`* / *`POP`* at the **end** of the array (*`slot 999,999`*) a 1-step *`O(1)`* operation, whereas removing from *`slot 0`* and sliding all *`999,999`* remaining items one slot left takes *`999,999`* steps (*`O(n)`*)?"
- Example good answer: Adding or removing at the end only touches one slot (`top`), whereas shifting every item left touches every remaining element in the array. (To make a Queue `O(1)` for both `ENQUEUE` and `DEQUEUE`, languages use a circular buffer with a moving `front` index or a doubly linked list like Python's `collections.deque`.)

✓ 5. Closing check (5 min)

1. Start with an empty stack `S`. Execute: `PUSH(10)`, `PUSH(20)`, `POP()`, `PUSH(30)`, `PUSH(40)`, `POP()`. What value is returned by the second `POP()`, and what items remain in `S` from bottom to top?

2. Start with an empty queue `Q`. Execute: `ENQUEUE(10)`, `ENQUEUE(20)`, `DEQUEUE()`, `ENQUEUE(30)`, `ENQUEUE(40)`, `DEQUEUE()`. What value is returned by the second `DEQUEUE()`, and what items remain in `Q` from front to back?
3. Trace the bracket-matching algorithm on `( [ ) ]`. What is on the stack right before the first closing bracket `)` is read, and why does the check fail?

Worked answers for closing check

1. First `POP()` removes `20` (leaving `[10]`). Then `30` and `40` are pushed (`[10, 30, 40]`). The second `POP()` returns **`40`**, leaving **`[10, 30]`** (`10` at bottom, `30` at top).
2. First `DEQUEUE()` removes `10` (leaving `[20]`). Then `30` and `40` are enqueued (`[20, 30, 40]`). The second `DEQUEUE()` returns **`20`**, leaving **`[30, 40]`** (`30` at front, `40` at back).
3. Right before `)` is read, the stack holds **`[ '(', '[' ]`** (with `'['` on top). When `)` is read, `POP()` returns `'['`, which does not match `')'`, so the algorithm immediately reports a mismatch failure.

Differentiation

- **Extra support:** Have students write the index number (`1, 2, 3...`) on each card as it enters and physically stack cards vertically for a Stack vs. slide cards horizontally through a paper tunnel for a Queue.
- **Extra challenge:** Show how a browser's **Back** and **Forward** buttons use **two stacks** (`back_stack` and `forward_stack`): clicking **Back** pops the current page from `back_stack` and pushes it onto `forward_stack`, while visiting a brand-new link clears `forward_stack`.

Teacher notes

- **Common misconception:** Students sometimes think a stack "sorts" items by size. Emphasize that stacks and queues care **only about arrival time**, never about whether `10` is smaller or larger than `40`.

Lesson 2: Finding Needles Without Scanning the Haystack — Hash Tables and Collisions

- **Unit:** Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)
- **Time:** 50 minutes
- **Grouping:** Whole class and pairs

Learning objective

Students will compute array bucket indices using a deterministic **hash function** (`h(k) mod M`), build a 7-bucket **hash table** with **separate chaining** to resolve collisions, and compare the number of comparisons needed for an unsorted list scan vs. a hash table lookup.

Materials

- Printed or whiteboard 7-bucket hash table grid (`Buckets 0 through 6`)
- Alphabet position strip (`A=1, B=2, C=3, ..., Z=26`) for quick arithmetic hashing by hand

Vocabulary

- **Key–Value Pair:** A unique identifier (**key**, such as a student ID or word) stored alongside its associated data (**value**, such as a grade or definition).
- **Hash Function (`h(k)`):** A deterministic math rule that converts any key `k` into an integer bucket index `0, 1, ..., M - 1`.
- **Collision:** When two different keys (`k1 != k2`) produce the **exact same bucket index** (`h(k1) == h(k2)`).
- **Separate Chaining:** Storing a short linked list (chain) of key–value pairs inside each bucket so colliding keys can share a bucket without overwriting each other.
- **Load Factor (`α = n / M`):** The ratio of stored items (`n`) to total buckets (`M`), connecting directly to Grade 6 Mathematics: Ratios.

Lesson flow

✎ 1. Opening: Why Not Just Scan a List? (8 min)

1. Suppose a school library database stores `n = 70,000` book barcodes in an unsorted list.
2. To check if barcode `B-94821` is checked out, a **linear scan** checks item 1, item 2, item 3... taking `35,000` comparisons on average and `70,000` comparisons in the worst case (`O(n)`).
3. Ask: "What if, instead of searching every shelf, we had a math formula that looked at the letters of a key and told us the exact shelf number in 1 step?"
4. That is a **Hash Table** (called a `dict` in Python or `Map` in JavaScript).

2. Direct instruction: Computing a Toy Hash Function & Modulo Indexing (15 min)

1. Define a simple, hand-computable hash rule for a table with `M = 7` **buckets** numbered `0` through `6`:
2. **Step A:** Add the alphabet positions (`A=1, B=2, ..., Z=26`) of the letters in the key (`sum`).
3. **Step B:** Divide `sum` by `7` and take the **remainder**: `bucket = sum mod 7` (the remainder is always `0, 1, 2, 3, 4, 5, or 6`!).
4. Work through 6 animal keys together on the board:

Key	Letter Values	sum	Division by 7	bucket = sum mod 7
"CAT"	3 + 1 + 20	24	24 = 3 × 7 + 3	**Bucket 3**
"DOG"	4 + 15 + 7	26	26 = 3 × 7 + 5	**Bucket 5**
"ANT"	1 + 14 + 20	35	35 = 5 × 7 + 0	**Bucket 0**
"OWL"	15 + 23 + 12	50	50 = 7 × 7 + 1	**Bucket 1**
"BEE"	2 + 5 + 5	12	12 = 1 × 7 + 5	**Bucket 5 (COLLISION with "DOG"!)**
"FOX"	6 + 15 + 24	45	45 = 6 × 7 + 3	**Bucket 3 (COLLISION with "CAT"!)**

1. Show why **Separate Chaining** is essential:
2. When `"BEE"` lands in **Bucket `5`**, we must **not** erase `"DOG"`. Instead, Bucket `5` holds a short chain: `Bucket 5 → [ ("DOG", val), ("BEE", val) ]`.
3. When looking up `"BEE"`, the computer computes `12 mod 7 = 5`, jumps straight to Bucket `5`, checks `"DOG"` (not a match), and finds `"BEE"` on the 2nd comparison—still only **2 comparisons** instead of scanning all 6 items!

✎ 3. Main activity: Pair Hash-Table Build & Worst-Case Adversarial Attack (17 min)

1. **Part A — Insert 3 More Keys (10 min):** Student pairs draw the 7-bucket table (`0..6`) with the 6 animals above already placed, then compute and insert three more keys:
2. `"ELK"` (`5 + 12 + 11 = 28`; `28 mod 7 = 0` → chains after `"ANT"` in **Bucket `0`**)
3. `"HEN"` (`8 + 5 + 14 = 27`; `27 mod 7 = 6` → lands in empty **Bucket `6`**)
4. `"COD"` (`3 + 15 + 4 = 22`; `22 mod 7 = 1` → chains after `"OWL"` in **Bucket `1`**)
5. **Part B — Count Comparisons & Load Factor (7 min):**
6. Total items `n = 9`, total buckets `M = 7` → **Load factor `α = 9 / 7 ≈ 1.29` items per bucket**.
7. What is the maximum number of key comparisons needed to find any of the 9 animals in our hash table? (Only **2 comparisons**, because the longest chain has length 2!)
8. **Adversarial thought experiment:** Why doesn't our hash function only check the first letter of a word? (Because in an English dictionary, thousands of words start with `'S'`, piling into one giant chain while `'X'` is nearly empty!)

✓ 4. Discussion questions (5 min)

- **Question 1:** "Why must a hash function be 100% deterministic (never using a random number generator each time you look up a key)?"

- Example good answer: If `h("CAT")` returned Bucket `3` when inserting and a random Bucket `6` when looking it up later, you would look in the wrong bucket and never find `"CAT"` again!
- **Question 2:** "What happens to lookup speed if we put $n = 7{,}000$ items into only $M = 7$ buckets ($\alpha = 1{,}000$)? How do real hash tables prevent that?"
- Example good answer: Each bucket chain would average `1,000` items long, slowing lookups back down to linear scans. Real hash tables **resize (rehash)**—doubling `M` whenever the load factor `α = n / M` passes a threshold like `0.75`.

✓ 5. Closing check (5 min)

Using the same rule (`letter sum mod 7`, with `A=1 ... Z=26`):

1. Compute the letter sum and bucket index (`0..6`) for the key **`"BAT"`** (`B=2, A=1, T=20`).
2. Does **`"BAT"`** collide with any key in our 9-animal table from the lesson? If so, which bucket does it join, and which keys are already in that bucket?
3. A hash table has `M = 50` buckets and currently stores `n = 35` keys. What is its **load factor** **`α`**, and why is a low load factor desirable?

Worked answers for closing check

1. `B(2) + A(1) + T(20) = 23`. Dividing by 7 gives `23 = 3 × 7 + 2`, so `23 mod 7 = 2` (**Bucket `2`**).
2. **No collision!** In our 9-animal table, Buckets `0, 1, 3, 5, 6` had items, while **Bucket `2`** and **Bucket `4`** were empty. So `"BAT"` is the first item in Bucket `2`.
3. **Load factor `α = 35 / 50 = 0.70`**. A low load factor means most buckets have 0 or 1 item, keeping lookups close to 1 comparison (`O(1)`).

Differentiation

- **Extra support:** Provide a printed multiplication table for `7` (`0, 7, 14, 21, 28, 35, 42, 49, 56, 63, 70`) so students can find `sum mod 7` immediately by subtracting the largest multiple of `7` at or below `sum`.
- **Extra challenge:** Have students find an **anagram collision** (such as `"ACT"`, `"CAT"`, and `"TAC"`): why do all anagrams collide under an addition-only hash function, and how does multiplying each letter by a position weight (`1×L1 + 31×L2 + 31²×L3`) break anagram collisions?

Teacher notes

- **Common misconception:** Students often assume a hash collision means the hash table "failed" or overwrote the earlier key. Emphasize that collisions are normal and expected (by the Pigeonhole Principle / Birthday Paradox), and that separate chaining stores both keys safely in the same bucket list.

Lesson 3: Modeling Networks — Graphs, Adjacency Lists, and Adjacency Matrices

- **Unit:** Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)
- **Time:** 50 minutes
- **Grouping:** Whole class and pairs

Learning objective

Students will model real-world systems as **graphs** `G = (V, E)` (choosing directed vs. undirected and weighted vs. unweighted edges), convert a graph between its **adjacency matrix** and **adjacency list** representations, and compare their memory and lookup trade-offs.

Materials

- Grid paper or printed worksheets for drawing a 5×5 adjacency matrix and adjacency list

Vocabulary

- **Graph (`G = (V, E)`):** A mathematical structure made of a set of **vertices** (nodes, `V`) and a set of **edges** (connections, `E`) linking pairs of vertices.
- **Directed vs. Undirected Edge:** An **undirected** edge `{u, v}` is a two-way connection (`u ↔ v`); a **directed** edge `(u → v)` is a one-way arrow from `u` to `v`.
- **Degree of a Vertex:** The number of edges connected to that vertex (in a directed graph, split into **in-degree** and **out-degree**).
- **Adjacency Matrix:** A `|V| × |V|` grid where row `u`, column `v` stores `1` (or edge weight `w`) if edge `(u, v)` exists, and `0` otherwise.
- **Adjacency List:** A dictionary/array where each vertex `u` stores a list of only its direct neighbors.

Lesson flow

✎ 1. Opening: Four Different Worlds, One Same Shape (8 min)

1. Put four scenarios on the board and ask what they have in common:
2. **Packet routers** connected by fiber-optic cables (Grade 6 Computing)
3. **Meadow habitat patches** connected by wildlife movement corridors (Grade 10 Science: Ecology Field Sampling)
4. **Web pages** connected by clickable hyperlinks (Grade 8 Society: Information Literacy)
5. **Cities** connected by rail lines
6. In every case, we have **nodes (vertices)** and **connections (edges)**. Once we represent any of these as a **graph**, the exact same algorithms can analyze all four!
7. Ask: "Which of those four has **one-way (directed)** edges by default?"
8. **Web hyperlinks!** Page `A` can link to Page `B` (`A → B`) without Page `B` linking back to Page `A`.

2. Direct instruction: Adjacency Matrix vs. Adjacency List (15 min)

Consider this 5-vertex **undirected** regional network with vertices `V = {A, B, C, D, E}` and 6 edges: `E = { {A,B}, {A,C}, {B,C}, {B,D}, {C,D}, {D,E} }`:

```
   A --- B
   |   / |
   |  /  |
   C --- D --- E
```

Build both computer representations side by side:

Representation 1: 5×5 Adjacency Matrix (`|V|² = 25` cells)

From \ To	A	B	C	D	E	Row Sum (Degree)
A	0	1	1	0	0	2
B	1	0	1	1	0	3
C	1	1	0	1	0	3
D	0	1	1	0	1	3
E	0	0	0	1	0	1

- Notice that for an **undirected** graph, the matrix is **symmetric across the main diagonal** (`matrix[u][v] == matrix[v][u]`), and the sum of all `1`s in the matrix (`12`) equals `2 × |E|` (the Handshaking Lemma: every undirected edge adds `1` to the degree of both its endpoints!).

Representation 2: Adjacency List (`|V| + 2|E|` entries)

```
graph = {
    "A": ["B", "C"],
    "B": ["A", "C", "D"],
    "C": ["A", "B", "D"],
    "D": ["B", "C", "E"],
    "E": ["D"]
}
```

✎ 3. Main activity: Sparse vs. Dense Trade-Off Analysis & Directed Conversion (17 min)

1. **Part A — Directed Graph Practice (9 min):** Pairs are given a 4-page mini-web of directed hyperlinks:
2. Page `1` links to `2` and `3` (`1 → 2`, `1 → 3`)
3. Page `2` links to `3` (`2 → 3`)
4. Page `3` links to `1` and `4` (`3 → 1`, `3 → 4`)
5. Page `4` links to `2` (`4 → 2`) Students write:
6. The 4×4 **directed adjacency matrix** (and verify it is **not** symmetric: `matrix[1][2] = 1`, `matrix[2][1] = 0`).
7. The **in-degree** (column sum = incoming links) and **out-degree** (row sum = outgoing links) of every page.
8. **Part B — Memory Calculation for a Real Road Map (8 min):**
9. Suppose a country road map has `|V| = 100,000` intersections, and each intersection meets an average of `4` roads (`200,000` undirected edges).

10. **Adjacency Matrix size:** `100,000 × 100,000 = 10,000,000,000` (`10 billion` cells—over `99.996%` of them filled with `0`s!).
11. **Adjacency List size:** `100,000` vertex buckets holding `2|E| = 400,000` neighbor entries (`25,000×` fewer neighbor entries than the matrix, or `|V| + 2|E| = 500,000` total entries including the vertex headers—`20,000×` smaller overall!).

✓ 4. Discussion questions (5 min)

- **Question 1:** "When is an Adjacency Matrix actually better than an Adjacency List?"
- Example good answer: When the graph is small or **dense** (nearly every vertex connects to every other vertex), or when you frequently need to check "Does edge *(u, v)* exist?" in a single array lookup `matrix[u][v]` (`O(1)`).
- **Question 2:** "Why is the sum of the degrees of all vertices in any undirected graph always an **even number**?"
- Example good answer: Because every single edge connects two endpoints, adding `+2` to the total degree sum (`Sum of degrees = 2 × |E|`).

✓ 5. Closing check (5 min)

1. An undirected graph has `|V| = 6` vertices and `|E| = 8` edges. How many total cells are in its **adjacency matrix**, how many of those cells contain a `1`, and how many contain a `0`?
2. Look at this adjacency list for a **directed** graph:
3. `X: [Y, Z]`
4. `Y: [Z]`
5. `Z: []` What are the **out-degree** and **in-degree** of vertex `Z`?
6. Would you use an **adjacency matrix** or an **adjacency list** to store a social network with `1,000,000` users where the average user has `150` connections? Explain why using numbers.

Worked answers for closing check

1. A `6 × 6` adjacency matrix has **`36` total cells**. Since each of the `8` undirected edges places two `1`s in the matrix (`matrix[u][v]` and `matrix[v][u]`), **`16` cells contain `1`** and `36 − 16 =` **`20` cells contain `0`**.
2. Vertex `Z` has **out-degree `0`** (its outgoing list `Z: []` is empty) and **in-degree `2`** (both `X` and `Y` point to `Z`).
3. An **adjacency list**, because it stores only `1,000,000 × 150 = 150,000,000` connection entries (`~1.5 × 10⁸`), whereas an adjacency matrix would require `1,000,000 × 1,000,000 = 1,000,000,000,000` (`10¹²`, one trillion) cells—nearly all zeros.

Differentiation

- **Extra support:** Have students color-code each undirected edge `{u, v}` with a matching pair of highlighters in the matrix (`row u, col v` and `row v, col u`) so the diagonal symmetry and the `2 × |E|` count become visually immediate.

- **Extra challenge:** Ask students how the adjacency matrix and adjacency list change if we allow **self-loops** (an edge from a vertex to itself, like a web page linking to itself) and **parallel edges** (two separate roads between the same two towns).

Teacher notes

- **Common misconception:** When reading a directed adjacency matrix, students frequently swap row (`from`) and column (`to`). Keep the header label **`From (Row) \ To (Column)`** visible on the board so out-degree is always the row sum and in-degree is always the column sum.

Lesson 4: Expanding Ripples — Breadth-First Search (BFS) and Fewest-Hop Routing

- **Unit:** Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)
- **Time:** 50 minutes
- **Grouping:** Whole class and pairs

Learning objective

Students will execute **Breadth-First Search (BFS)** step by step on an unweighted graph using a FIFO **queue**, a `visited` set, a `dist` (hop count) table, and `parent` pointers to reconstruct the shortest unweighted path between two vertices.

Materials

- Printed 6-vertex network diagram (below) and index cards to serve as the FIFO queue

Vocabulary

- **Breadth-First Search (BFS):** A graph traversal algorithm that explores all neighbors at distance `1` hop from the start vertex before exploring vertices at distance `2` hops, `3` hops, and so on.
- **Visited Set:** A record of vertices already discovered (enqueued) so the algorithm never gets trapped in an infinite loop around a cycle.
- **Parent (Predecessor) Pointer:** For each discovered vertex `v`, the vertex `u` that first discovered `v` (`parent[v] = u`), allowing us to trace the shortest path backward from the goal to the start.

Lesson flow

✎ 1. Opening: Dropping a Stone in a Pond (7 min)

1. Consider this 6-router network (`A` through `F`) where every link takes 1 hop: `A --- B --- E | | | C --- D --- F` Edges (alphabetical neighbor order):
2. `A: [B, C]`
3. `B: [A, D, E]`
4. `C: [A, D]`
5. `D: [B, C, F]`
6. `E: [B, F]`
7. `F: [D, E]`
8. Ask: "If router *A* wants to send a packet to router *F* in the **fewest hops**, why can't a computer just wander down the first road it sees (*A → C → D → B → E → F*, 5 hops)?"
9. Explain the ripple idea: If we explore in expanding rings—**Ring 0** (`A`), **Ring 1** (all 1-hop neighbors of `A`), **Ring 2** (all 2-hop neighbors), **Ring 3** (all 3-hop neighbors)—then the very **first time** a ring touches `F`, we are guaranteed to have found a path with the minimum possible number of hops!

10. Ask: "Which data structure from Lesson 1 processes items in exact First-In, First-Out order so Ring 1 finishes before Ring 2?" → **A Queue (FIFO)!**

2. Direct instruction: Complete Step-by-Step BFS Trace from `A` to `F` (16 min)

Show the exact BFS algorithm and trace table (breaking neighbor ties alphabetically):

```
from collections import deque

def bfs_shortest_path(graph, start, goal):
    queue = deque([start])
    visited = {start}
    dist = {start: 0}
    parent = {start: None}

    while queue:
        u = queue.popleft()          # DEQUEUE front vertex
        if u == goal:
            break
        for v in graph[u]:           # Inspect each neighbor of u
            if v not in visited:
                visited.add(v)       # Mark visited IMMEDIATELY upon enqueue!
                dist[v] = dist[u] + 1
                parent[v] = u
                queue.append(v)      # ENQUEUE v at back
    return dist, parent
```

Trace every iteration on the board:

Step	Dequeued `u`	Unvisited Neighbors Enqueued	Queue (`front → back`)	`visited` Set	Newly Recorded `dist[v]` & `parent[v]`
Init	—	Start at `A`	`[A]`	`{A}`	`dist[A]=0`, `parent[A]=None`
1	**`A`**	`B`, `C`	`[B, C]`	`{A, B, C}`	`dist[B]=1 (p=A)`, `dist[C]=1 (p=A)`
2	**`B`**	`D`, `E` (`A` already in `visited`)	`[C, D, E]`	`{A, B, C, D, E}`	`dist[D]=2 (p=B)`, `dist[E]=2 (p=B)`
3	**`C`**	(none: *`A`* & *`D`* already in *`visited`*!)	`[D, E]`	`{A, B, C, D, E}`	—
4	**`D`**	`F` (`B` & `C` already in `visited`)	`[E, F]`	`{A, B, C, D, E, F}`	`dist[F]=3 (p=D)`
5	**`E`**	(none: *`B`* & *`F`* already in *`visited`*)	`[F]`	`{A, B, C, D, E, F}`	—
6	**`F`**	Goal reached!	`[]`	`{A, B, C, D, E, F}`	Stop

Reconstructing the path via `parent` pointers: Start at `F`: `parent[F] = D` → `parent[D] = B` → `parent[B] = A` → `parent[A] = None`. Reverse it: **`A → B → D → F`** (`3` hops!). (Note: `A → B → E → F` and `A → C → D → F` are also tied at `3` hops; alphabetical tie-breaking discovered `D` via `B` first, and `F` via `D` first.)

> **Critical bug to highlight:** Look at Step 2 and Step 3. Why did we mark `D` as `visited` the moment `B` **enqueued** `D` in Step 2, instead of waiting until `D` was dequeued in Step

4? Because in Step 3 (`u = C`), `C` also has an edge to `D`! If `D` were not already in `visited`, `C` would enqueue a duplicate copy of `D` into the queue. Always mark vertices `visited` **at enqueue time**.

✎ 3. Main activity: Pair BFS Trace on a 7-Node Campus Graph (17 min)

Student pairs trace BFS starting from `S` to find the hop distance and `parent` tree to all vertices on this 7-vertex graph (alphabetical tie-breaking): - `S: [A, B]` - `A: [S, C, D]` - `B: [S, D]` - `C: [A, G]` - `D: [A, B, E, G]` - `E: [D]` - `G: [C, D]`

Pairs fill in the complete `dist` and `parent` table and verify: - Layer 0: `S` (`dist=0`) - Layer 1: `A` (`dist=1, p=S`), `B` (`dist=1, p=S`) - Layer 2: `C` (`dist=2, p=A`), `D` (`dist=2, p=A`) - Layer 3: `G` (`dist=3, p=C`), `E` (`dist=3, p=D`) - Shortest path from `S` to `G`: **`S → A → C → G`** (3 hops).

✓ 4. Discussion questions (5 min)

- **Question 1:** "What happens to our algorithm if we replace the FIFO **Queue** (*`popleft()`*) with a LIFO **Stack** (*`pop()`*)?"
- Example good answer: It explores in a **depth-first** style instead of expanding in layer rings (`1 hop, 2 hops, 3 hops`), plunging deep down one branch before backtracking. Because it does not finish Layer 1 and Layer 2 before exploring deeper nodes, the first path it records is **not** guaranteed to be the shortest path (for example, on our 7-node campus graph, popping from the end visits `S, B, D, G, C` and records a `4`-hop path `S → B → D → G → C` to `C` instead of the `2`-hop shortest path `S → A → C`; on the 6-router network from `A`, it records a `4`-hop path `A → C → D → F → E` to `E` instead of `2` hops `A → B → E`).
- **Question 2:** "How many times does BFS enqueue each vertex and inspect each edge in a connected graph *`G = (V, E)`*?"
- Example good answer: Each vertex enters the queue at most **once** (`|V|` dequeues), and each edge is inspected at most **twice** (once from each endpoint), giving a linear time cost of **`O(|V| + |E|)`**.

✓ 5. Closing check (5 min)

1. In the 7-vertex campus graph (`S → A, B` above), both `A` and `B` have an edge to `D`. Why did `parent[D]` get set to `A` instead of `B`?
2. What would happen if you ran BFS on a graph containing a triangle cycle `A – B – C – A` **without** keeping a `visited` set?
3. After running BFS from `Start = M`, your `parent` table records: `parent[M] = None`, `parent[P] = M`, `parent[Q] = P`, `parent[R] = P`, `parent[T] = R`. What is the shortest path from `M` to `T`, and how many hops long is it?

Worked answers for closing check

1. Because `A` was enqueued before `B` (alphabetical order) and therefore dequeued first. When `A`'s neighbors were scanned, `D` was unvisited, so `D` was marked visited with `parent[D] = A`. When `B` was dequeued next, `D` was already in `visited`.
2. Without a `visited` set, vertices are enqueued repeatedly around the cycle (`A` enqueues `B, C`; `B` enqueues `A, C`; `C` enqueues `A, B`, and so on). If the code stops at a reachable `goal` (such as

`goal = C` from `A`), it still halts once `C` is dequeued (after 3 dequeues), but leaves duplicate entries in the queue and wastes exponential work on larger graphs; if we are exploring the whole graph (or the `goal` is unreachable), the queue grows forever and the algorithm **never terminates**.

3. Tracing backward from `T`: `T ← R ← P ← M`. Reversing gives **`M → P → R → T`**, which is **`3` hops** long.

Differentiation

- **Extra support:** Give pairs physical tokens or sticky notes labeled `"VISITED"` to place directly on each vertex the moment its card enters the queue, reinforcing that marking happens at **enqueue** time rather than dequeue time.
- **Extra challenge:** Have students trace true recursive or late-marked **Depth-First Search (DFS)** (where a vertex is marked visited when popped rather than when pushed) alongside our enqueue-marked stack variant on the 7-vertex campus graph and compare the two visitation orders.

Teacher notes

- **Common misconception:** Students often forget to reverse the `parent` pointer chain at the end (`F → D → B → A` vs. `A → B → D → F`). Remind them that `parent[v]` points **backward** toward the start vertex, just like breadcrumbs leading back home.

Lesson 5: Weighted Routes — Dijkstra's Shortest-Path Algorithm

- **Unit:** Data Structures, Graph Algorithms, and Shortest Paths (Computing, Grade 10)
- **Time:** 50 minutes
- **Grouping:** Whole class and pairs

Learning objective

Students will explain why fewest hops (BFS) can fail on a **weighted graph**, execute **Dijkstra's shortest-path algorithm** step by step using a tentative distance table (`dist[]`, `prev[]`, `settled`), and explain why negative edge weights violate Dijkstra's greedy settlement rule.

Materials

- Printed weighted graph trace sheets (diagrams below) and pencils

Vocabulary

- **Weighted Graph:** A graph where each edge `(u, v)` has a numerical **weight (cost)** `w(u, v)` representing distance, travel time, cable cost, or network latency.
- **Tentative Distance (`dist[v]`):** The shortest known total cost from the start vertex to `v` found so far (initialized to `0` for `start` and `∞` for all other vertices).
- **Edge Relaxation:** Checking whether going from `start` through `u` to neighbor `v` (`dist[u] + w(u, v)`) beats the current best known cost `dist[v]`; if `dist[u] + w(u, v) < dist[v]`, we update `dist[v]` and set `prev[v] = u`.
- **Settled (Finalized) Vertex:** A vertex whose minimum distance from `start` is permanently locked in.

Lesson flow

✎ 1. Opening: When Fewest Hops Is the Slowest Route (7 min)

1. Draw a 3-node triangle on the board representing travel times in minutes between `A`, `B`, and `C`:
2. Direct highway edge `A — C` has weight **`10` minutes** (congested construction zone, `1` hop).
3. Local road `A — B` has weight **`2` minutes**, and `B — C` has weight **`3` minutes** (`2` hops total).
4. Ask:
5. "Which route from *`A`* to *`C`* does **BFS (Lesson 4)** pick?" → `A → C` (`1` hop, **10 minutes**), because BFS ignores edge weights and only counts hops!
6. "Which route is actually faster?" → `A → B → C` (`2 + 3 =` **5 minutes**!).
7. Introduce Edsger W. Dijkstra's 1959 insight: When all edge weights are **non-negative (`w >= 0`)**, if we always pick the **unsettled vertex `u` with the smallest tentative `dist[u]`**, no detour through any other unsettled vertex can ever make `dist[u]` smaller!

2. Direct instruction: Complete Step-by-Step Trace of Dijkstra's Algorithm (16 min)

Consider this 5-vertex weighted network (weights in milliseconds of network latency): - `A – B` (weight `4`), `A – C` (weight `1`) - `C – B` (weight `2`), `C – D` (weight `5`) - `B – D` (weight `1`), `B – E` (weight `7`) - `D – E` (weight `3`)

```
       (4)       (7)
   A ------- B ------- E
   |       / |       /
(1)|  (2)/   |(1)   /(3)
   |   /     |     /
   C ------- D ---/
       (5)
```

Dijkstra's Algorithm Rules: 1. Set `dist[A] = 0` and `dist[v] = ∞` for all other vertices. Set `settled = {}`. 2. While unsettled vertices remain: - **Pick `u`:** Choose the **unsettled** vertex with the **minimum `dist[u]`** and mark `u` as **settled**. - **Relax edges out of `u`:** For each unsettled neighbor `v` of `u`, compute `new_cost = dist[u] + w(u, v)`. If `new_cost < dist[v]`, update `dist[v] = new_cost` and `prev[v] = u`.

Trace every iteration on the board:

Round	Settled `u` (`dist[u]`)	Edge Relaxations Tested from `u`	`dist[A]`	`dist[B]` (prev)	`dist[C]` (prev)	`dist[D]` (prev)	`dist[E]` (prev)	Settled Set
Init	—	—	`0`	`∞`	`∞`	`∞`	`∞`	`{}`
1	**`A` (0)**	`A→B: 0+4=4 < ∞ ✓` `A→C: 0+1=1 < ∞ ✓`	**`0`**	`4 (A)`	`1 (A)`	`∞`	`∞`	`{A}`
2	**`C` (1)**	`C→B: 1+2=3 < 4 ✓` **(improved!)** `C→D: 1+5=6 < ∞ ✓`	`0`	**`3 (C)`**	**`1 (A)`**	`6 (C)`	`∞`	`{A, C}`
3	**`B` (3)**	`B→D: 3+1=4 < 6 ✓` **(improved!)** `B→E: 3+7=10 < ∞ ✓`	`0`	`3 (C)`	`1 (A)`	**`4 (B)`**	`10 (B)`	`{A, C, B}`
4	**`D` (4)**	`D→E: 4+3=7 < 10 ✓` **(improved!)**	`0`	`3 (C)`	`1 (A)`	`4 (B)`	**`7 (D)`**	`{A, C, B, D}`
5	**`E` (7)**	(all neighbors settled)	`0`	`3 (C)`	`1 (A)`	`4 (B)`	**`7 (D)`**	`{A, C, B, D, E}`

Reconstruct the minimum-weight path from `A` to `E`: Follow `prev` backward from `E`: `E ← D ← B ← C ← A`. Reverse it: **`A → C → B → D → E`** with total cost `1 + 2 + 1 + 3 = 7 ms`! Notice that `A → B → E` (2 hops) costs `4 + 7 = 11 ms`, and `A → C → D → E` (3 hops) costs `1 + 5 + 3 = 9 ms`, while the **4-hop path `A → C → B → D → E` costs only `7 ms`**!

✎ 3. Main activity: Pair Dijkstra Trace & The Negative-Weight Counterexample (17 min)

1. **Part A — Pair Dijkstra Trace (12 min):** Pairs trace Dijkstra's algorithm from `S` to `T` on this 5-vertex graph:
2. `S – A` (6), `S – B` (2)

3. `B – A (3)`, `B – C (8)`
4. `A – C (1)`, `A – T (9)`
5. `C – T (4)` Students fill out the 5-round table and verify:
6. Round 1 settles `S (0)` → `dist[A]=6 (S)`, `dist[B]=2 (S)`
7. Round 2 settles `B (2)` → `B→A` relaxes `dist[A]` from `6` down to `2+3 = 5 (B)`; `B→C` sets `dist[C] = 2+8 = 10 (B)`
8. Round 3 settles `A (5)` → `A→C` relaxes `dist[C]` from `10` down to `5+1 = 6 (A)`; `A→T` sets `dist[T] = 5+9 = 14 (A)`
9. Round 4 settles `C (6)` → `C→T` relaxes `dist[T]` from `14` down to `6+4 = 10 (C)`
10. Round 5 settles `T (10)` → Shortest path is **`S → B → A → C → T`** with cost **`10`**.
11. **Part B — Why Negative Edges Break Dijkstra (5 min):** Draw a 3-vertex directed graph:
12. `X → Y` has weight `5`
13. `X → Z` has weight `2`
14. `Y → Z` has weight **`-6`** (a negative edge!)
15. Trace what Dijkstra does from `X`:
    - Round 1 settles `X (0)` → `dist[Z] = 2`, `dist[Y] = 5`.
    - Round 2 greedily settles **`Z (2)`** because `2 < 5`!
    - But going `X → Y → Z` actually costs `5 + (-6) = -1 < 2`! Because `w(Y, Z) = -6` was negative, Dijkstra's assumption that "adding more edges can never reduce cost" failed.

✓ 4. Discussion questions (5 min)

- **Question 1:** "What happens when you run Dijkstra's algorithm on a graph where every single edge has weight *`w = 1`*?"
- Example good answer: It behaves identically to **Breadth-First Search (BFS)**! BFS is simply the special case of Dijkstra's algorithm when all edge weights are equal to `1`.
- **Question 2:** "In our Lesson 5 network (*`A → C → B → D → E`*), *`dist[B]`* was first set to *`4`* in Round 1, and then lowered to *`3`* in Round 2. Why was it legal to change *`dist[B]`* in Round 2?"
- Example good answer: Because `B` was still **unsettled** in Round 2! A vertex's distance is only tentative until that vertex itself is chosen as the minimum unsettled vertex `u`.

✓ 5. End-of-unit synthesis & Closing check (5 min)

1. In the Part A trace (`S, A, B, C, T`), why did Dijkstra settle vertex `B` (`dist = 2`) before vertex `A` (`dist = 6`) in Round 2?
2. Give a concrete 3-vertex example (`P, Q, R` with positive edge weights) where the **1-hop path `P → R`** is longer than the **2-hop path `P → Q → R`**.
3. Which data structure or algorithm from this 5-lesson unit (`Stack`, `Queue`, `Hash Table`, `BFS`, or `Dijkstra`) should an engineer choose for each task below?
4. **(a)** Looking up a user's profile by their unique username `"alex_42"` in `O(1)` average time.
5. **(b)** Finding the fastest driving route across a highway network where each road segment is labeled with its travel time in minutes.
6. **(c)** Implementing the `Undo` button in a drawing program.

7. **(d)** Finding the minimum number of friend-of-a-friend introductions connecting two people in an unweighted social graph.

Worked answers for closing check

1. Because at the start of Round 2, the unsettled vertices had tentative distances `dist[B] = 2`, `dist[A] = 6`, `dist[C] = ∞`, and `dist[T] = ∞`. Dijkstra always selects the unsettled vertex with the **smallest tentative distance** (`2 < 6`).
2. Any triangle where `w(P, Q) + w(Q, R) < w(P, R)`, such as `w(P, R) = 15`, `w(P, Q) = 4`, and `w(Q, R) = 5` (`4 + 5 = 9 < 15`).
3. **(a) Hash Table**; **(b) Dijkstra's Algorithm**; **(c) Stack (LIFO)**; **(d) Breadth-First Search (BFS)** (using a FIFO **Queue**).

Differentiation

- **Extra support:** Have students cross out settled vertices with a check mark (✓) in the table header at the end of each round so they only compare tentative distances among the remaining unsettled columns.
- **Extra challenge:** Ask students how A (A-Star) Search* speeds up Dijkstra's algorithm on a physical map by adding an admissible heuristic estimate `h(v)` (such as straight-line Euclidean distance to the goal) to the priority key `dist[v] + h(v)`.

Teacher notes

- **Common misconception:** Students sometimes try to add the new edge weight `w(u, v)` to the old `dist[v]` instead of computing `dist[u] + w(u, v)`. Write the relaxation question on the board in words: "Is going through *u* (*dist[u] + edge*) shorter than the best route to *v* we knew so far (*dist[v]*)?"