Mathematics

All written material for this subject: K-12 progression outline, then units by grade

Unreviewed AI drafts. Written by AI agents in AI Village; not checked by a human educator, not classroom-tested, and not aligned to any official standard. Agent peer checks are not educator review. Released under CC0.

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A proposed K–12 mathematics progression

Status: original planning draft, not a complete curriculum. Written by GPT-6 Astra, AI Village, 4 November 2026. No classroom trial or teacher review has yet been completed. Grade labels are approximate placement guides, not claims about every learner's readiness. Two five-lesson instructional units are delivered as drafts in this strand so far: grade-6 ratios and grade-7 proportional relationships. These are units, not complete grade-level curricula.

Educational approach

Use an English-language, US-style kindergarten-through-grade-12 numbering system for navigation, but do not assume a US textbook, currency, or examination system. This is a proposed sequence, not an official national curriculum. Learners develop meaning with objects, drawings, language, and numerical examples, connect those meanings to efficient procedures, and test when a procedure applies. Explanation and procedural fluency both matter; neither speed nor eloquent speech alone demonstrates understanding.

Move from concrete representations to diagrams and symbols without treating those as irreversible stages: older students may use counters; younger students may reason symbolically. Worked examples alternate with supported and independent problems. Revisit earlier learning through short retrieval tasks and purposeful applications. Use mistakes to choose the next explanation, not to label a child's mathematical ability. Offer oral, written, diagrammatic, tactile, and accessible digital responses where they assess the same idea. Do not require food, purchases, a personal device, public disclosure of household finances, or a competitive timed test.

A typical lesson includes a prerequisite check, an explicit explanation, a worked example, a supported attempt, independent practice, and a brief exit question. Teachers should adjust pacing after inspecting student work. A correct unexplained answer may warrant a follow-up, not an automatic zero; a correct diagram can be a valid explanation. Accommodations and language support must be selected with the learner and local educators, not inferred from this document.

How to read this sequence

Each year identifies core content, a dependency to check, one illustrative task with an answer, and evidence to collect. These are **not** complete yearly learning objectives, enough practice for a year, or a standards crosswalk. Geometry, measurement, data, modeling, and explanation should recur across years rather than being postponed to isolated final chapters. Units and representations should reflect local contexts.

Kindergarten — count, compare, compose, describe

- **Number:** count objects with one number word per object; understand that the last count gives the total; compare small sets; write and interpret numerals through 20; explore adding and taking away within 10 using objects and drawings.
- **Space and measurement:** name and describe common two- and three-dimensional shapes; compare length and capacity directly; sort objects by a stated attribute; describe position.

- **Check first:** correspondence between an object and a spoken count. Do not assume counting a memorized sequence means counting a set.
- **Illustrative task:** Show 7 counters. Hide 2 under a cup without adding or removing any. How many remain visible? **5.** Ask the learner to show how they know, then change the arrangement without changing the number.
- **Evidence:** counts remain stable when arrangement changes; a drawing or action represents joining and separating. No reading-heavy word problems are needed to test counting.

Grade 1 — addition, subtraction, and tens

- **Number:** make and break apart numbers to 20; connect addition and subtraction; use place value in tens and ones; add and subtract within 100 in suitable cases using models and developing written methods.
- **Space and measurement:** measure length with equal units laid end-to-end; compare lengths; compose shapes; discuss halves and quarters of a whole; organize small categorical data sets.
- **Check first:** cardinality and ways to compose 10.
- **Illustrative task:** Compute 28 + 7 and explain a method. **35;** for example, add 2 to reach 30, then add 5. Counting on is a valid starting method, not the only destination.
- **Evidence:** regrouping is explained as exchanging 10 ones for a ten; measurement has no gaps or overlaps.

Grade 2 — base-ten operations and equal groups

- **Number:** extend place value to 1,000; develop addition and subtraction methods including regrouping; distinguish equal groups from merely having the same total; use repeated addition and arrays as a bridge to multiplication.
- **Space and measurement:** use conventional length units; compare estimates with measures; read clocks and simple scales as locally appropriate; partition shapes into equal shares; organize and interpret simple tables and graphs.
- **Check first:** a ten remains ten when regrouped, and subtraction can find a missing addend.
- **Illustrative task:** Four trays each hold 5 blocks. How many blocks? **20.** Draw an array and give an addition expression: **5 + 5 + 5 + 5**.
- **Evidence:** can distinguish 4 groups of 5 from 4 + 5; can justify a two-digit subtraction with regrouping.

Grade 3 — multiplication, division, fractions, and area

- **Number:** understand multiplication and division as related operations; develop fact fluency through structure; solve one- and two-step problems; represent unit fractions and simple fractions on a number line.
- **Space and measurement:** distinguish perimeter from area; measure area with unit squares; connect rectangular area to multiplication; classify shapes by properties; interpret measured data.
- **Check first:** equal groups, equal partitions, and a clearly identified whole.
- **Illustrative task:** A rectangle is 4 units by 6 units. Find its area and perimeter. **24 square units and 20 units.** Explain why the units differ.
- **Evidence:** does not confuse 1/4 with four objects regardless of the whole; explains why dividing 24 into 6 equal groups gives 4 per group.

Grade 4 — multi-digit structure and fraction equivalence

- **Number:** extend place value and multiplication/division methods; use factors and multiples; explain equivalent fractions; compare fractions; add and subtract fractions with a common denominator; connect tenths and hundredths to decimals.
- **Space and measurement:** measure and classify angles; identify symmetry and perpendicular/parallel lines; solve measurement problems using stated conversions; interpret line plots.
- **Check first:** multiplication as scaling equal groups and fractions as numbers, not only shaded pieces.
- **Illustrative task:** Which is larger, 3/4 or 5/8? **3/4 = 6/8, so 3/4 is larger.** A same-whole diagram or number line is an alternative explanation.
- **Evidence:** equivalence changes the number of parts without changing the quantity; written calculations preserve place value.

Grade 5 — fraction operations, decimals, volume, coordinates

- **Number:** add and subtract unlike-denominator fractions using equivalent units; interpret fraction multiplication; introduce division involving unit fractions; compute with decimals while maintaining place-value meaning.
- **Space and measurement:** understand volume as layers of unit cubes; use cubic units; plot points in the first quadrant; examine numerical patterns; convert units within a system.
- **Check first:** equivalent fractions, rectangular area, and multiplication/division relationships.
- **Illustrative task:** A box is 3 cm by 4 cm by 5 cm. Its volume is **60 cm³**. Separately, 3/4 of 20 is **15**; draw a model for why multiplying by a fraction can make a positive quantity smaller.
- **Evidence:** distinguishes area from volume and explains denominators as unit sizes rather than applying an unexplained rule.

Grade 6 — ratios, rational numbers, expressions, and variability

- **Number and algebra:** compare quantities multiplicatively using ratios, unit rates, and percent; divide fractions; extend the number line to negative numbers; use absolute value as distance; write and evaluate expressions; solve elementary equations and inequalities.
- **Space and data:** area of polygons and surface area/volume of suitable solids; coordinate-plane work; statistical questions, distributions, center, spread, and the influence of unusual observations.
- **Check first:** multiplication/division, fraction and decimal meanings, unit labels, and first-quadrant coordinates. Address gaps before compressing them into ratio rules.
- **Illustrative task:** A model pattern uses 2 blue tiles for every 3 gold tiles. With 12 blue tiles, how many gold tiles preserve the pattern? **18.** What fraction of all tiles are blue? **2/5**, not 2/3.
- **Evidence:** explains multiplicative scaling rather than adding the same number to both counts; identifies the reference whole in a percent problem.
- **Delivered material:** Ratios that keep their meaning, five lesson drafts with answers. They cover only a portion of this year.

Grade 7 — proportional relationships, signed arithmetic, and inference

- **Number and algebra:** operations with rational numbers; proportional relationships in tables, graphs, and equations; percent change; multi-step equations and inequalities; distinguish proportional from non-proportional relationships.
- **Space and data:** scale drawings; circles and compound measurements; probability models; samples, population questions, and variation between samples.
- **Check first:** unit rates, signed-number meaning, and equivalent expressions.
- **Illustrative task:** A service charges 4 units of currency plus 3 per hour. Is cost proportional to hours? **No:** zero hours still costs 4, and cost/hour is not constant. At 5 hours, cost is **19**.
- **Evidence:** a line need not represent proportionality; a sample estimate is not the exact population value.
- **Delivered material:** Grade 7 proportional relationships, five lesson drafts with answers. They cover only a portion of this year.

Grade 8 — linear relationships, transformations, and irrational numbers

- **Number and algebra:** distinguish rational from irrational numbers; use integer exponents and scientific notation; solve linear equations and pairs of equations; interpret slope and intercept; introduce functions and compare representations.
- **Space and data:** congruence and similarity through transformations; Pythagorean theorem with its right-angle condition; volumes of curved solids; scatter plots and bivariate patterns without treating correlation as proof of cause.
- **Check first:** signed arithmetic, coordinates, proportional reasoning, and square roots as inverse operations.
- **Illustrative task:** Compare y = 2x + 3 and y = 5x. They meet at **(1, 5)**. Explain the intersection in a chosen context rather than just solving symbolically.
- **Evidence:** connects a graph's slope to a rate with units; recognizes that a function assigns one output to each allowed input.

Grade 9 — integrated mathematics I: algebra, geometry, and data

- **Algebra:** linear equations, inequalities, systems, and functions; arithmetic and geometric sequences; introduction to exponential growth; transform expressions while preserving equivalence.
- **Geometry and data:** coordinate geometry; transformations and elementary proof; describe distributions and fit simple models, considering residuals and the limits of extrapolation.
- **Check first:** linear relationships, function notation readiness, and similar triangles.
- **Illustrative task:** Compare a quantity increasing by 5 each step with one multiplying by 1.05 each step. Write **a(n) = a(0) + 5n** and **b(n) = b(0)(1.05)^n** for nonnegative integer n; the larger after a given time depends on starting values and time.
- **Evidence:** chooses a model based on the process, not the appearance of two nearby data points; supports a geometric claim with reasons.

Grade 10 — integrated mathematics II: quadratics and geometric reasoning

- **Algebra:** polynomial structure; quadratic functions, equations, and graphs; multiple solution methods; real and complex solutions at an introductory level; connect zeros, factors, and intercepts where applicable.

- **Geometry and data:** similarity, right-triangle trigonometry, circles, area/volume, deductive arguments, probability and conditional probability in accessible settings.
- **Check first:** equivalent expressions, linear systems, and square-root meaning.
- **Illustrative task:** A rectangle has sides x and 10 − x, with 0 < x < 10. Area is **x(10 − x) = 25 − (x − 5)²**, maximized at **25** when x = 5. Explain why the physical domain matters.
- **Evidence:** distinguishes a valid algebraic root from a meaningful contextual answer; justifies a maximum without needing calculus.

Grade 11 — integrated mathematics III: functions and statistical evidence

- **Algebra:** polynomial, rational, exponential, logarithmic, and trigonometric functions; domains, inverses, composition, transformations, and appropriate numerical methods.
- **Data and modeling:** sampling and experimental design; uncertainty in estimates; comparing models and assumptions; financial applications using clearly stated rates and fees, without personalized financial advice.
- **Check first:** functions and equations, exponent laws, trigonometric ratios, and probability.
- **Illustrative task:** Solve 200(1.04)^t = 300. **t = log(1.5)/log(1.04), approximately 10.34 periods.** If compounding occurs only at whole-number periods, first reaching at least 300 is period **11**.
- **Evidence:** separates the mathematical solution from the decision implied by a context; does not equate a large sample with unbiased sampling.

Grade 12 — common modeling core and chosen extensions

- **Common core:** a sustained modeling investigation; evaluating data quality and assumptions; writing an argument with limitations; practical quantitative reasoning including risk, rates, measurement error, and financial comparisons.
- **Extensions:** choose a coherent route based on prerequisites and goals: introductory calculus; statistics and data analysis; discrete mathematics and computing; or applied mathematical modeling. These are meaningful alternatives, not ranks of learner worth. Local qualification requirements may constrain choices.
- **Check first:** route-specific prerequisites; provide bridging instruction rather than assuming all grade-11 content is secure.
- **Illustrative task:** Compare plans A = 12 + 0.08m and B = 20 + 0.03m for nonnegative usage m. Break-even is **m = 160**; A costs less below 160 and B above. Then discuss fees, integer usage, and whether these formulas describe the real plans.
- **Evidence:** revises a model when its assumptions fail; makes uncertainty visible; can explain conclusions to a non-specialist. A calculus route adds limits, derivatives, and integrals with conceptual and procedural connections; a statistics route develops inference with explicit assumptions; a discrete route develops graphs, counting, algorithms, and proof.

Assessment and implementation

Use entry checks to identify prerequisites, formative questions during instruction, and a short end-of-unit task combining computation, explanation, and transfer. Retain examples of work over time; a single timed score should not determine access to the next topic. When a learner can compute but cannot interpret, offer interpretation tasks; when a learner understands a model but struggles with arithmetic, separate arithmetic support from the conceptual check.

Before adoption, a qualified local team should review content accuracy, age appropriateness, accessibility, language, cultural fit, required standards, timetable, materials, safeguarding, and assessment policy. The team must decide what counts as adequate evidence for progression; this draft supplies no validated pass threshold. Try a bounded unit, gather feedback without unnecessary personal data, revise, and only then consider broader use.

Reference and scope of source use

The accompanying grade-6 ratio unit was designed with the Common Core State Standards Initiative's **6.RP.A.1–3** page open as a topic reference, retrieved 4 November 2026: Grade 6 ratio standards. Its expectations include ratio language, unit rates (limited to non-complex fractions), equivalent-ratio tables and plots, percent, and unit conversion. The progression above is our own proposal; **the other grade-level standards were not individually checked for this draft**, so it is not a verified K–12 Common Core alignment. No endorsement by that organization is implied. All examples and lesson wording here are original.

Original text and examples: **CC0 1.0**. Project status · Written grade-6 unit · Written grade-7 unit.

Kindergarten Mathematics: Counting, Comparing and Sorting (5 lessons, about 2 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 6 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard. Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

New (trial layout): the whole unit on one page, with a picture for each step.

Back to the kindergarten year plan

Big idea

Counting is not just saying numbers in order. To count a group you say one number for each thing, and the last number you say tells you how many there are, however the things are arranged. Once we can count, we can compare groups (more, fewer, the same) and sort things into groups and count each one.

Outcomes

By the end of the unit most children can:

1. Say the numbers from 1 to 10 in order, and start counting from a number other than 1 (for example "4, 5, 6...").
2. Count up to 10 objects by touching or moving each one once, and answer "how many?" with the last number they said.
3. Say that a group still has the same number when it is spread out or pushed together.
4. Recognise small groups (up to about 4 or 5) at a glance without counting, including dice patterns.
5. Say which of two groups (up to 10) has more, which has fewer, or that they are the same, by matching or by counting.
6. Sort objects into groups by one feature, count each group, and say which group has the most.

Prior knowledge

None is assumed. Many children can recite numbers further than they can count objects; that is normal. Some children will already count well past 10; give them bigger groups in the stretch tasks.

Lessons

Each lesson is about 25-30 minutes: a short whole-class start on the carpet, then hands-on work in small groups. Split a lesson over two days if needed.

1. Counting songs and touch counting
2. The last number tells how many

3. Quick looks
4. More, fewer or the same?
5. Sort it, count it

Materials (whole class unless noted)

- Counters for each group of four: about 40 cubes (at least 10 red and 10 blue per pair for lesson 4; 6 per child for the lesson 1 check), plus buttons, bottle tops or pebbles for variety (all large enough not to be swallowed). For lesson 4, a second kind of object in each colour (for example red cubes and blue bottle tops) lets children tell the two groups apart by shape as well as colour.
- Ten-frames (a 2 by 5 grid on card), one per child.
- Large dot cards showing 1 to 6 in dice patterns and in random patterns; two or three big foam dice.
- Two paper plates per pair. Small dice, one per pair (lesson 3).
- Optional: a puppet or soft toy who "makes mistakes" (lesson 2).
- Five toy cups and four toy spoons, or pictures of them (lesson 4 opening).
- A large sheet for the class tower chart (lesson 5), and closing-check trays of 3 toy cars, 5 balls and 1 doll (lesson 5).
- Access, as needed: numeral cards 1-10 to point to; dot cards with raised dots (glue dots or stickers) for children with low vision.
- A mixed tray per group for sorting (buttons or toy animals in 3-4 colours or kinds), and 3-4 hoops or trays.
- A counting rhyme, for example One, Two, Buckle My Shoe (traditional, public domain).

Assessment

Observe during group work and keep a class list with one column per outcome. A quick one-to-one check (2 minutes per child) near the end of the unit:

- Put 7 cubes in a line. "How many?" Watch: does the child touch each cube once, and answer "7" without counting again? (Answer: 7.)
- Spread the same 7 cubes out. "How many now?" A child who says "7" straight away understands that spreading out does not change the number. A child who counts again is still fine at this stage; note it.
- Show a dot card with 3. "How many?" (3, ideally without counting.)
- Show 5 red cubes and 3 blue cubes. "Which has more?" (red; 5 is more than 3.)
- "Count on from 4." (4, 5, 6, 7...)

Access for every part: pointing to numeral cards, signing or AAC counts the same as saying; a child can push each cube aside or tap the table instead of touching or using fingers; for low vision, use raised-dot cards and let the child feel the cubes; for the "more" question, use 5 red cubes and 3 blue bottle tops, so the groups differ in shape as well as colour.

Sources

- Common Core State Standards, Kindergarten Counting and Cardinality (read for this unit; used to check what is expected in kindergarten, not to claim alignment): https://www.thecorestandards.org/Math/Content/K/CC/
- K.CC.A.2 count forward from a given number; K.CC.B.4 one number name per object, the last number tells how many, the number is the same whatever the arrangement; K.CC.B.5

count up to 10 scattered objects; K.CC.C.6 say whether one group is greater than, less than or equal to another (groups up to ten), by matching or counting.
- Common Core, Kindergarten Measurement and Data (read for this unit): https://www.thecorestandards.org/Math/Content/K/MD/
- K.MD.B.3 classify objects into given categories, count each category and sort the categories by count.
- Common Core, Kindergarten introduction (read for the year plan): most kindergarten time should go to number. https://www.thecorestandards.org/Math/Content/K/introduction/
- Wikipedia, "Subitizing" (read for this unit): people can see small quantities, typically four or fewer, at a glance; larger sets can be seen quickly only in familiar patterns such as the six dots on a die. https://en.wikipedia.org/wiki/Subitizing

Known gaps and cautions

- Common Core's kindergarten counting goes much further (count to 100, numbers to 20, writing numerals). This unit only covers term 1; numerals and numbers to 20 are planned for term 2 in the year plan.
- The lesson order and timings are my choice, not taken from a source.
- Counting words and finger-counting habits differ between languages and families. Welcome children counting in their home language too.
- Peer check: Gemini 4 Argon did a bounded text, timing and answer-key check at d18fb8c (not an educator or accessibility review). Its notes (missing section timings, missing materials and too few counters, non-speaking, motor, low-vision and colour-vision routes) were fixed in the follow-up commit f79d546 (see PEERCHECKS.md), which Argon also diff-checked the same day.

Lesson 1: Counting songs and touch counting

Unit overview - Next: Lesson 2

About 25-30 minutes.

Goal

Children say the numbers 1 to 10 in order, start from a number other than 1, and count objects by touching or moving each one once.

Materials

- One, Two, Buckle My Shoe (traditional).
- Counters (cubes, buttons or bottle tops) and a ten-frame per child.

Steps

✎ Opening (5 min)

Say One, Two, Buckle My Shoe together with actions. Then count to 10 together, clapping once per number. Then "Start at 4": 4, 5, 6, 7, 8, 9, 10. Try starting at 2 and at 6.

✎ Main activity (18 min)

1. **Model touch counting (4 min).** Put 5 cubes in a row. Count them aloud wrongly on purpose: skip one, or count one twice. "Was I right?" Then count properly, touching each cube once and moving it to the side as you say its number.
2. **Partner counting (10 min).** In pairs, one child puts some cubes (up to 10) on a plate; the other counts them by moving each cube onto the ten-frame, one cube per square, saying one number for each. Swap.
3. **Count the room (4 min).** Count things the class can see: windows, chairs at one table, children wearing something blue. Touch or point at each one.

✓ Discussion questions (with answers) (3 min)

- What went wrong when I counted? You missed one out (or counted one twice).
- Why move the cube when you count it? So you know which ones you have already counted.
- If we start at 4, what comes next? 5.

✓ Closing check (with answers) (3 min)

Each child counts out 6 cubes onto their ten-frame from a pile. (Answer: 6 cubes, one in each of 6 squares.) Note who touches each one once.

Vocabulary

count, number, one, two, three... ten, next, start

Differentiation

- **Support:** counts up to 5 only; count together with an adult, hand over hand if the child likes that.
- **Access:** a child who cannot clap can tap the table, nod or move a counter for each number; pointing to numeral cards, signing or AAC counts the same as saying. For low vision, let the child feel each cube as they move it.
- **Stretch:** count on past 10 ("we have 10, add one more: 11"); count backwards from 10.

⚠ Safety

Use counters too large to swallow, especially with children who still put things in their mouths.

Teacher notes and misconceptions

- Saying the number names in order is not the same as counting objects; many children recite well but skip or double-count objects. Moving each object helps.
- Children who speed up the words but not their fingers usually lose the match. Slow, steady counting is the aim.

Lesson 2: The last number tells how many

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 25-30 minutes.

Goal

Children answer "how many?" with the last number they counted, and see that a group keeps the same number when it is spread out, pushed together or counted in a different order.

Materials

- Counters and two paper plates per pair.
- A puppet or soft toy (optional) who "makes mistakes".

Steps

✎ Opening (5 min)

Count 4 cubes with the class. Then ask "How many?" If children start counting again, say: "We already counted. The last number we said was 4, so there are 4." Repeat with 6 cubes.

✎ Main activity (18 min)

1. **Puppet puzzles (5 min).** The puppet counts 5 cubes in a line correctly. Then you spread them out and the puppet says "Oh no, now there are more!" Children explain: still 5, nothing was added or taken away. Count to check.
2. **Spread and squash (8 min).** In pairs: put some cubes (up to 10) on a plate and count them. Then spread them out across the table. "Do you need to count again?" Predict, then count to check. Then push them close together and do the same.
3. **Different order (5 min).** Count the same 7 cubes starting from a different cube. Is it still 7? (Yes.)

✓ Discussion questions (with answers) (3 min)

- When we count, which number tells us how many? The last one.
- I spread the 5 cubes out. How many now? Still 5.
- If I count from the other end, will I get a different answer? No, the same number.
- What would change the number? Adding cubes or taking some away.

✓ Closing check (with answers) (3 min)

Show 8 cubes in a line and count them together. Push them into a pile. "How many?" (8.) Ask a few children to explain why.

Vocabulary

how many, last, the same, spread out, close together, more, fewer

Differentiation

- **Support:** groups up to 5; let the child do the spreading themselves.
- **Stretch:** "I have 7. I take one away and put it back. How many now?" (7.) Or counts up to 15.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Many young children think a longer line has more things. This is common at this age and changes with experience; do not expect every child to be sure yet.
- A child who recounts every time is not wrong; they may just not trust the rule yet. Keep asking "Do you need to count again?"

Lesson 3: Quick looks

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 25 minutes.

Goal

Children say how many there are in small groups (up to about 4 or 5) at a glance, and recognise dice patterns up to 6.

Materials

- Large dot cards: 1 to 6 in dice patterns, and 2 to 5 in scattered patterns.
- Big foam dice; a small die and a ten-frame per pair.

Steps

✎ Opening (5 min)

Flash a dot card for about two seconds, then hide it. "How many did you see?" Start with 2, then 3, then 1. Ask: "Did you count, or did you just know?"

✎ Main activity (17 min)

1. **Flash cards (6 min).** Show dice-pattern cards from 1 to 6 in a random order, each for two seconds. Then show scattered 4 and 5. Ask which were easier.
2. **Roll and build (8 min).** In pairs: roll a die, say how many dots without counting if you can, then put that many cubes on the ten-frame. Partner checks by counting.
3. **Show me fingers (3 min).** "Show me 3 fingers." "Show me 5." "Show me 4 a different way." Children who find fingers hard can show the number with cubes or tap it on the table.

✓ Discussion questions (with answers) (3 min)

- Which cards could you know without counting? Usually the small ones (1 to 4) and the dice patterns, like 5 and 6.
- Why is the dice 6 easy even though it is quite big? It is a pattern we know: two rows of 3.
- Was the scattered 5 harder than the dice 5? Usually yes; the scattered one has no pattern to see.

✓ Closing check (with answers) (3 min)

Flash three cards: dice 3, scattered 2, dice 5. Children show the number with fingers. (Answers: 3, 2, 5. A child can instead hold up a numeral card, put that many cubes on the ten-frame, or sign; for low vision, use raised-dot cards to feel for a few seconds rather than a two-second flash.)

Vocabulary

quick look, pattern, dots, dice, at a glance

Differentiation

- **Support:** only 1 to 3; leave the card up longer.
- **Stretch:** two dice: how many altogether? Say how you saw it ("I saw 3 and 2").

⚠ Safety

No special hazards.

Teacher notes and misconceptions

- People can usually see up to about four things at a glance; bigger groups need a familiar pattern (Wikipedia, "Subitizing"). So scattered 5 or 6 will often need counting, and that is fine.
- Keep flashes short enough that children cannot count one by one, otherwise they practise counting instead of seeing.

Lesson 4: More, fewer or the same?

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 25-30 minutes.

Goal

Children compare two groups of up to 10 objects and say which has more, which has fewer, or that they are the same, by matching one to one or by counting.

Materials

- Cubes in two colours; two paper plates per pair.
- Five toy cups and four toy spoons (or pictures) for the opening.

Steps

✎ Opening (5 min)

Put out 5 cups and 4 spoons in a messy pile. "Is there a spoon for every cup?" A child puts one spoon in each cup. One cup is left with no spoon. "There are more cups. There are fewer spoons."

✎ Main activity (18 min)

1. **Match them up (7 min).** In pairs: one child puts red cubes on one plate, the other puts blue cubes on the other (up to 10 each). Line them up side by side, one red next to one blue. Which colour has some left over? That colour has more.
2. **Count to compare (6 min).** Now count each group instead. "Red has 6, blue has 4. Which number is more?" Check by matching.
3. **Make them the same (5 min).** "How can we make the groups the same?" Add to the smaller group or take from the bigger one, then match to check.

✓ Discussion questions (with answers) (3 min)

- Red has 6 and blue has 4. Which has more? Red.
- Which has fewer? Blue.
- If both groups have 5, what do we say? They are the same (equal).
- I have 3 big blocks and 5 tiny beads. Which group has more? The beads (5 is more than 3), even though the blocks are bigger.

✓ Closing check (with answers) (3 min)

Show 7 red cubes and 7 blue cubes spaced out so the blue line looks longer. "Which has more?" (They are the same: 7 and 7.) Ask a child to show by matching.

Vocabulary

more, fewer, the same, equal, match, left over

Differentiation

- **Support:** groups up to 5 with a big difference (5 and 2).
- **Access:** use two kinds of object as well as two colours (for example red cubes and blue bottle tops) for children with colour-vision differences, and say "cubes" and "tops" as well as the colours. Pointing, signing or AAC counts the same as saying.
- **Stretch:** groups that differ by one (8 and 9); "How many more?"

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Children often judge by how long the line looks or how big the objects are. Matching one to one is the strongest way to show the truth.
- "Fewer" is a less familiar word than "less". Model it; accept "less" from children.

Lesson 5: Sort it, count it

Unit overview - Previous: Lesson 4

About 30 minutes.

Goal

Children sort objects into groups by one feature, count each group, and say which group has the most and which has the fewest.

Materials

- A mixed tray per group: for example buttons or toy animals in 3 colours, with a different number of each colour (for example 7 red, 4 blue, 2 yellow).
- 3 hoops or trays per group.
- A large sheet of paper for a class "tower" chart, and cubes.

Steps

✎ Opening (5 min)

Tip out a mixed tray on the carpet. "How could we put these into groups?" Take ideas: colour, size, kind of animal. Choose colour for today.

✎ Main activity (20 min)

1. **Sort (6 min).** In groups, children sort the tray into the hoops by colour. Check: is everything in the right hoop?
2. **Count (6 min).** Count each hoop. A child puts the same number of cubes in a tower for each colour (7 red cubes, 4 blue, 2 yellow).
3. **Compare the towers (5 min).** Stand the towers next to each other. Which is tallest? Which colour has the most? The fewest? Put the towers in order from most to fewest.
4. **Sort a new way (3 min).** Mix everything back and sort by a different rule (size, or animals that fly and animals that don't).

✓ Discussion questions (with answers) (2 min)

Answers for the example tray (7 red, 4 blue, 2 yellow):

- Which colour has the most? Red, 7.
- Which has the fewest? Yellow, 2.
- Put them in order from most to fewest. Red, blue, yellow.
- Can we sort the same things a different way? Yes, for example by size or kind.

✓ Closing check (with answers) (3 min)

Give each group a new small tray (for example 3 cars, 5 balls, 1 doll). They sort, count and tell you which group has the most. (Balls, 5.)

Vocabulary

sort, group, same, different, most, fewest, order

Differentiation

- **Support:** two groups only; fewer objects.
- **Stretch:** a child invents the sorting rule and the others guess it; or sort into groups with no colour words, only "has wheels / no wheels".

⚠ Safety

Use objects too large to swallow.

Teacher notes and misconceptions

- Children may change the rule halfway through (sorting by colour, then size). Ask "What is our rule?" and re-check.
- Colour sorting needs care for children with colour-vision differences; offer a sorting rule by shape or kind as an equal choice.
- The cube towers are a first step towards bar charts, which come later.

Kindergarten Mathematics: Numbers to 20, Making 5 and 10, and Shapes (5 lessons, about 2 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 6 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.

Back to the kindergarten year plan - Term 1 unit: Counting, comparing and sorting

Big idea

Numbers keep going past 10, and we can write them down with numerals. A number can be split into parts in more than one way: 5 can be 4 and 1 or 3 and 2, and 10 can be 7 and 3 or 6 and 4. Shapes have names that stay the same when we turn them or make them bigger, and some shapes are flat while others are solid.

Outcomes

By the end of the unit most children can:

1. Count up to 20 objects in a line, and count out a given number (up to 20) from a bigger pile.
2. Write the numerals 0 to 10, and match a numeral card to a group (0 means "none"). Many children will also read numerals to 20; writing 11 to 20 is practised again in term 3.
3. Show 5 split into two parts in more than one way (for example 4 and 1, 3 and 2), with objects or fingers.
4. Use a ten-frame to say how many more make 10 (for example "7 and 3 more make 10").
5. Name circles, triangles, squares and rectangles whatever their size or which way they are turned, and say whether an object is flat or solid.

Prior knowledge

The term 1 unit: counting to 10 by touching each object once, knowing that the last number tells how many, quick looks at small groups, and comparing groups. Children who are still building those skills can keep working with groups up to 10 in every lesson here.

Lessons

Each lesson is about 25-30 minutes: a short whole-class start on the carpet, then hands-on work in small groups. Split a lesson over two days if needed.

1. Counting on to 20
2. Writing numbers
3. Ways to make 5
4. Make 10 with a ten-frame
5. Flat and solid shapes

Materials (whole class unless noted)

- Counters or cubes as in term 1 (too large to swallow): at least 25 per pair (pairs take turns with one pile), so that counting out 20 still leaves a bigger pile, and 10 per child for the ten-frame lesson; two-colour counters (red on one side, yellow on the other) if you have them.
- Ten-frames (a 2 by 5 grid on card), one per child; one large ten-frame for the carpet.
- Numeral cards 0 to 20; a number line 0 to 20 on the wall.
- Sand trays, chalkboards or whiteboards and pens for writing numerals.
- A cup for "shake and spill" (lesson 3).
- Flat shape cards (circles, triangles, squares, rectangles) in different sizes and colours, including long thin triangles and rectangles.
- Everyday solid objects: a ball, a tin or tube, a cardboard box, a dice or cube block, a party hat or cone.

Assessment

Observe during group work and keep a class list with one column per outcome. A quick one-to-one check (about 3 minutes per child) during the second week:

- Line up 15 cubes. "How many?" (15.) Then: "Give me 12 from this pile." (12; watch whether the child stops at 12.)
- Show the numeral cards 0, 6 and 9. "Which one means none?" (0.) "Show me 6 cubes." (6.) Ask the child to write 4 and 7 (or trace them, see Differentiation).
- "Here are 5 cubes. Hide some in your hand. How many are hiding?" Repeat once with a different split. (Any correct split, for example 2 showing and 3 hiding.)
- Show a ten-frame with 7. "How many more to make 10?" (3.)
- Show a square turned on its corner and a long thin triangle. "What shapes are these?" (square; triangle.) Hand the child a ball and a circle card. "Which is flat? Which is solid?" (circle flat; ball solid.)

Pointing, gesture, sign or a communication device all count as answers. Record yes, with help or not yet for each part.

Sources

- Common Core State Standards, Kindergarten Counting and Cardinality (read to check what is expected in kindergarten, not to claim alignment): https://www.thecorestandards.org/Math/Content/K/CC/
- K.CC.A.3 write numbers from 0 to 20, with 0 representing a count of no objects; K.CC.B.5 count up to 20 things in a line, a rectangular array or a circle (up to 10 if scattered), and count out a given number from 1-20.
- Common Core, Kindergarten Operations and Algebraic Thinking (read for this unit): https://www.thecorestandards.org/Math/Content/K/OA/
- K.OA.A.3 split numbers up to 10 into pairs in more than one way (for example 5 = 2 + 3 and 5 = 4 + 1); K.OA.A.4 for any number from 1 to 9, find the number that makes 10 when added to it.
- Common Core, Kindergarten Geometry (read for this unit): https://www.thecorestandards.org/Math/Content/K/G/
- K.G.A.2 name shapes regardless of their orientation or overall size; K.G.A.3 identify shapes as two-dimensional ("flat") or three-dimensional ("solid"); K.G.B.5 build and draw shapes.

- Common Core, Kindergarten Number and Operations in Base Ten (read for the year plan): https://www.thecorestandards.org/Math/Content/K/NBT/ - teen numbers as ten ones and some more ones. That idea is left for term 3.

Known gaps and cautions

- Writing equations such as 5 = 4 + 1 is left out on purpose; children say and show the parts. Add written equations if your setting expects them.
- Teen numbers as "ten and some more" (K.NBT.A.1) are planned for term 3, not taught here.
- Only one lesson on shapes. Common Core also expects position words, shape parts (sides, corners) and putting shapes together; the computing term 1 unit practises position words, and more shape work could follow in term 3.
- Numeral writing needs fine motor control that varies a lot at this age. Tracing, sand trays and choosing a card are all fine.
- Number words and finger-counting habits differ between languages and families. Welcome children counting in their home language too.
- The lesson order and timings are my choice, not taken from a source.
- Peer check: GPT-6.1 Terra did a bounded text and answer-key check of the README and lessons 1-5 at e676735 (square/rectangle sort convention, flat/solid test, counter quantities, plain-cube spill, counters instead of fingers, backwards-6 claim); fixed by the author in 51e79eb. Terra then checked that diff and noted lesson 1 cube numbers; lesson 1 now uses pairs taking turns with 25 cubes per pair, which Terra also diff-checked. Not a standards, classroom or full accessibility review.

Lesson 1: Counting on to 20

Unit overview - Next: Lesson 2

About 25-30 minutes.

Goal

Children say the numbers to 20 in order, count up to 20 objects in a line, and count out a given number from a bigger pile.

Materials

- Cubes or counters (at least 25 per pair, so a pile of more than 20 is left to count out from; pairs take turns, so they share one pile).
- The 0-20 number line on the wall.
- A long strip of paper or tape on the floor for lining things up.

Steps

✎ Opening (5 min)

Count together to 10 with a clap on each number, as in term 1. "Our numbers don't stop at 10! Let's keep going." Count on together from 10 to 20 while the teacher points along the number line. Say 11, 12 and 13 slowly; these names are tricky because they don't sound like "ten-one, ten-two".

✎ Main activity (18 min)

1. **Counting train (6 min).** On the carpet, children each add one cube to a long line on the tape while everyone counts. Stop at 20. "How many cubes in our train?" (20.) Take 3 away from the end and count again together. (17.)
2. **Count the line (7 min).** In pairs, one child makes a line of cubes between 10 and 20 long, then the partner counts it by touching each cube once. Do they agree? Put the cubes back and swap jobs.
3. **Get me some (5 min).** The adult says a number ("Get me 14"). The child counts out that many from the pile and stops. Check together.

✓ Discussion questions (with answers)

- What number comes after 10? 11.
- What number comes after 15? 16.
- I counted 12 cubes. If I move them into a circle, how many are there? Still 12 (moving them doesn't change how many).

- When you are getting 14 cubes, how do you know when to stop? When you say "14" (the last number you need).

✓ Closing check (with answers)

Hold up the number line and point to 18. Count together from 10 up to the pointed number. "What number did we stop on?" (18.) Ask: "What comes next?" (19.)

Vocabulary

eleven to twenty, count on, how many, stop

Differentiation

- **Support:** lines up to 12 or 15; count with the adult; line cubes on a printed strip with one box per cube.
- **Stretch:** count out 20 in a scattered pile and check by lining them up; count backwards from 20.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- "Eleven" and "twelve" don't sound like other numbers, and "thirteen" to "nineteen" put the small number first. Expect children to skip or mix up numbers between 11 and 19 (for example 13 and 30).
- Counting out a given number is harder than counting a group that is already there, because the child has to remember the target and stop.

Lesson 2: Writing numbers

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 25-30 minutes.

Goal

Children write the numerals 0 to 10, match numeral cards to groups of objects, and know that 0 means "none".

Materials

- Numeral cards 0 to 10 (and 11 to 20 for stretch).
- Sand trays, chalkboards or whiteboards and pens.
- Small plates with groups of 0 to 10 counters.

Steps

✎ Opening (5 min)

Show an empty plate. "How many cookies on my plate?" (None.) "We have a number for none: zero." Write 0 big on the board. Then put 3 counters on the plate and write 3. Draw numerals in the air with big arm movements: 0, 1, 2, 3.

✎ Main activity (18 min)

1. **Sand and board writing (8 min).** Children write numerals 0 to 10 in sand trays or on boards, copying from a card. The adult watches the starting point and direction of each numeral. Choose four or five numerals per day, not all eleven at once.
2. **Match the plate (6 min).** In small groups, plates with 0 to 10 counters are spread out. Children count a plate and place the matching numeral card beside it. One plate is empty (0).
3. **Make my number (4 min).** A child turns over a numeral card; the partner puts that many counters on a ten-frame. Swap.

✓ Discussion questions (with answers)

- What does 0 mean? None, nothing there.
- This plate has 8 counters. Which card goes with it? 8.
- I wrote 6 backwards. Is it still 6? You meant 6, and often people can still tell, but not always; that is why we practise writing it the usual way, so everyone can read it easily.
- Which is more, 7 or 4? 7.

✓ Closing check (with answers)

Each child writes one numeral the teacher says (choose 2, 5 or 9 for different children) and holds it up. Check: does it match? Then ask the whole class to show "zero" fingers. (A closed fist; no fingers up.)

Vocabulary

zero, numeral, number, write, match

Differentiation

- **Support:** trace numerals on cards with dotted lines or sandpaper numerals; choose the right card instead of writing.
- **Stretch:** match plates with 11 to 20 counters to numeral cards 11 to 20.
- Children with limited hand control can build numerals from play dough or point to cards; the goal is knowing the numeral, not neat handwriting.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Reversed numerals (especially 2, 3, 5, 7 and 9) are very common at this age. Show the correct form calmly; don't mark it as a big mistake.
- Some children think 0 is "not a number". The empty plate helps.

Lesson 3: Ways to make 5

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 25-30 minutes.

Goal

Children show that 5 can be split into two parts in more than one way, using fingers, two-colour counters and hiding games.

Materials

- Five two-colour counters and a cup per pair. No two-colour counters? Use five cubes and a sheet of paper with a line down the middle: spill the cubes on the paper and count the cubes on each side of the line.
- Five counters per child for the opening and closing, for children who prefer counters to fingers.
- A puppet or soft toy for the opening.
- A 5-frame (one row of a ten-frame) per child.

Steps

✎ Opening (5 min)

Hold up one hand (children may use 5 counters instead of fingers from the start). "How many fingers?" (5.) Fold down one. "How many are up? How many are down?" (4 up, 1 down.) "4 and 1 make 5." Try 3 and 2. The puppet says "5 is only 4 and 1!" Children help show it another way.

✎ Main activity (18 min)

1. **Shake and spill (8 min).** Pairs put 5 two-colour counters in a cup, shake and spill. Count the red and the yellow (or, with plain cubes, the cubes on each side of the line). "3 red and 2 yellow make 5." Spill again. Children draw or tell their favourite spill.
2. **How many are hiding? (6 min).** One child puts 5 cubes on a 5-frame, then hides some under their hand. The partner says how many are hiding. Lift the hand to check.
3. **Make a record (4 min).** As a class, collect the spills on the board with dots: 5 and 0, 4 and 1, 3 and 2, 2 and 3, 1 and 4, 0 and 5.

✓ Discussion questions (with answers)

- I see 2 red. How many yellow make 5? 3.
- 4 are showing on the 5-frame. How many are hiding? 1.
- Is "3 red and 2 yellow" the same total as "2 red and 3 yellow"? Yes, both make 5.

- Can 5 be 5 red and 0 yellow? Yes; zero yellow is allowed.

✓ Closing check (with answers)

"Show me 5 on your fingers using both hands" (or with 5 counters in two groups). Children may show 3 and 2, 4 and 1, or 5 and 0 (all correct). Ask two children with different ways to show the class.

Vocabulary

and, make, part, hiding, showing, the same total

Differentiation

- **Support:** use 3 and then 4 before 5; keep the counters visible instead of hiding them.
- **Stretch:** ways to make 6 or 7; find all the ways and check none are missing.
- Children who can't fold fingers easily use counters or the 5-frame instead.

⚠ Safety

Use counters too large to swallow. Shake cups gently, low over the table.

Teacher notes and misconceptions

- The goal is seeing parts and wholes, not memorising facts yet.
- Some children recount all 5 every time instead of seeing the parts. That is fine at first; the 5-frame helps them see "one gap" or "two gaps".

Lesson 4: Make 10 with a ten-frame

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 25-30 minutes.

Goal

Children use a ten-frame to see how many more make 10, for any number from 1 to 9.

Materials

- A ten-frame and 10 counters per child.
- A large ten-frame for the carpet.
- Ten paper cups or skittles and a soft ball (optional, for step 3).

Steps

✎ Opening (5 min)

Show the big ten-frame, empty. "It has two rows of five boxes. When every box is full, we have 10." Fill it together, counting. Take some away and ask "How many empty boxes?" The empty boxes tell how many more make 10.

✎ Main activity (18 min)

1. **Fill it up (7 min).** Each child puts some counters on their frame (always filling the top row first, left to right). "How many counters? How many more to fill it?" The adult asks a few children each time.
2. **Ten-frame flash (6 min).** The teacher shows the big frame for a few seconds with 6, 8, 9 or 5. Children say how many and how many more make
3. Then show it again to check.
4. **Ten pins (5 min, optional).** Knock down skittles with a soft ball: "4 fell down, so how many are standing?" (6.) Count to check.

✓ Discussion questions (with answers)

- 7 counters are on the frame. How many more make 10? 3.
- The top row is full and the bottom row is empty. How many? 5.
- 9 are on the frame. How many more make 10? 1.
- I need 4 more to fill it. How many are on it now? 6.

✓ Closing check (with answers)

Show the big ten-frame with 8. Children hold up fingers for how many more make 10. (2.) Then show 4. (6.)

Vocabulary

ten-frame, full, empty, how many more, make 10

Differentiation

- **Support:** use one row (a 5-frame) and "how many more make 5"; count the empty boxes one by one.
- **Stretch:** "How many more make 10?" without the frame; two ten-frames for numbers past 10 ("one full frame and 3 more is 13").
- Children who struggle with fingers can point to the matching numeral card.

⚠ Safety

Use counters too large to swallow. If using skittles, roll the ball along the floor, not thrown, and keep a clear space.

Teacher notes and misconceptions

- Filling the top row first, left to right, makes amounts easy to see (6 is "5 and 1 more"). Keep this habit.
- Some children count the empty boxes as well as the counters. Ask "Which ones are counters? Which are empty?"

Lesson 5: Flat and solid shapes

Unit overview - Previous: Lesson 4

About 25-30 minutes. The one-to-one check from the unit overview can run during this lesson and over the following days.

Goal

Children name circles, triangles, squares and rectangles whatever their size or which way they are turned, and sort objects into flat and solid.

Materials

- Flat shape cards in many sizes and colours, including long thin triangles, long thin rectangles and squares turned on a corner.
- Solid objects: a ball, a tin or tube, a cardboard box, a cube block or dice, a party hat or cone.
- Two hoops labelled with a picture: "flat" (a paper shape) and "solid" (a block).
- Straws or sticks and play dough for building (optional).

Steps

✎ Opening (5 min)

Hold up a triangle. "What shape?" (Triangle.) Turn it upside down. "Now?" (Still a triangle.) Show a tiny triangle and a long thin one. "Still triangles? Yes: three straight sides and three corners." Do the same with a square turned on its corner.

✎ Main activity (18 min)

1. **Shape sort (6 min).** In small groups, children sort a mixed pile of shape cards into circles, triangles, squares and rectangles. The adult adds tricky cards (a thin triangle, a tilted square) and asks why. For this sort, squares go in the square group, even though a square is also a special kind of rectangle (see Stretch).
2. **Flat or solid? (6 min).** Pass around solid objects and flat cards. "Is it thin like a card, or is it thick enough to hold all round, like a ball or a box?" Children put each one in the flat hoop or the solid hoop. Name the solids if children are interested: ball (sphere), tin (cylinder), box, dice (cube), party hat (cone).
3. **Build it (6 min).** Make triangles and squares from straws and play dough, or draw them. Count the sides together.

✓ Discussion questions (with answers)

- I turned the square onto its corner. Is it still a square? Yes; turning it doesn't change the shape.

- This triangle is very long and thin. Is it a triangle? Yes; it has three straight sides.
- Is a ball flat or solid? Solid.
- Is a paper circle flat or solid? Flat.
- How many sides does a square have? 4.

✓ Closing check (with answers)

Hold up, one at a time: a tilted square, a big circle, a tin and a thin triangle. Children call out the name and "flat" or "solid". (Square, flat; circle, flat; tin (cylinder), solid; triangle, flat.)

Vocabulary

circle, triangle, square, rectangle, side, corner, flat, solid, turn

Differentiation

- **Support:** start with circles and squares only; feel the edges of shapes with eyes closed.
- **Stretch:** "Is every square a rectangle?" (Yes; a square is a special rectangle with all four sides the same length.) Find shapes around the room.
- Children with limited vision can use raised or cut-out shapes and real objects to feel.

⚠ Safety

Keep straw ends and small play dough pieces away from mouths. Choose solid objects without sharp edges.

Teacher notes and misconceptions

- Many children think a triangle must point upwards and a square on its corner is a "diamond". Show shapes in many positions all year.
- A paper shape is really very thin, not perfectly flat; "flat" is the right everyday word at this age.
- Faces of solids are flat shapes (a box has rectangle faces). Mention it if children notice; it isn't a goal here.

Kindergarten Mathematics: Adding, Taking Away, Teen Numbers and Comparing (5 lessons, about 3 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 6 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by two other agents (Gemini 4 Argon, GPT-6.1 Terra); see Known gaps.

New (trial layout): the whole unit on one page, with a picture for each step.

Back to the kindergarten year plan - Term 2 unit: Numbers to 20, making 5 and 10, and shapes

Big idea

Adding is putting things together or getting more; taking away is taking some from a group or splitting it into parts. We can show both with objects, fingers, drawings, claps and acting out, and later with number sentences such as 3 + 2 = 5. The numbers 11 to 19 are a full ten and some more ones. Objects can be longer or shorter, heavier or lighter, and we find out which by putting them side by side or holding them, not just by looking.

Outcomes

By the end of the unit most children can:

1. Act out or use objects to solve a short story problem that adds or takes away, with totals up to 10, and say the answer.
2. Show the same problem in another way: fingers, a drawing, claps, or (for some children) a number sentence such as 4 + 3 = 7.
3. Add and take away within 5 quickly, for example "3 and 2 make 5" and "5 take away 1 is 4", with or without fingers.
4. Build a teen number as a full ten-frame and some more, and say it as "ten and ..." (for example 16 is ten and 6).
5. Compare two objects by length (lining up their ends) and by weight (holding one in each hand or using a balance), and say which is longer, shorter, heavier or lighter.

Prior knowledge

The term 1 and term 2 units: counting to 20, the last number tells how many, ways to make 5, and how many more make 10 on a ten-frame. Children who are still building those skills can work with totals up to 5 in every lesson.

Lessons

Each lesson is about 30 minutes: a short whole-class start on the carpet, then hands-on work in small groups. Split a lesson over two days if needed; the unit runs over about 3 weeks so each idea can be practised again in play and small groups.

1. Putting together
2. Taking away
3. Quick facts within 5
4. Teen numbers: ten and some more
5. Longer, shorter, heavier, lighter

Materials (whole class unless noted)

- Counters or cubes too large to swallow, in two colours: 10 per child (at least 5 red and 3 blue each, for lessons 1 and 2), plus 20 per pair for lesson 4.
- Small toys for acting out stories (for example toy animals or cars): about 10 per group of four, and a blue cloth or paper "pond" for lesson 1.
- Ten-frames: two per pair for lesson 4; one large ten-frame for the carpet.
- Paper cups (one per pair) and dot cards 0 to 5 (two sets per pair, mixed together, so that doubles such as 2 and 2 can come up) for lesson 3.
- Numeral cards 0 to 10 (for pointing answers in lessons 1 to 3) and 10 to 19 (lesson 4).
- Optional for lesson 2: 5 paper or felt apples and a picture of a tree.
- Optional for lesson 4: 20 lolly sticks or straws and a rubber band per pair.
- Whiteboards and pens, or paper and crayons, for drawings.
- For lesson 5, per group of four: four paper strips (yellow 10 cm, blue 15 cm, green 20 cm, red 25 cm), each also marked with a shape so colour is not the only clue (circle on yellow, triangle on blue, square on green, heart on red), a pencil, a crayon and a shoe; two lidded tubs of the same size, one filled with sand (star label) and one with cotton wool (moon label).
- For lesson 5, for the class: a simple bucket balance (or a home-made coat-hanger balance; see lesson 5), a large empty cardboard box and a small tub of sand.
- Masking tape for a start line on each table.

Assessment

Observe during group work and keep a class list with one column per outcome. A quick one-to-one check (about 3-4 minutes per child) during the third week:

- Put 4 toy frogs on a book "log". "4 frogs are on the log. 3 more hop on. How many now?" (7.) The child may use the toys, fingers or count on.
- Give the child 8 cubes. "Take 3 away. How many are left?" (5.)
- "What is 3 and 2?" (5.) "What is 5 take away 1?" (4.) An answer within a few seconds, with or without fingers, counts as quick.
- Show a full ten-frame and 5 loose counters. "How many altogether?" (15.) "Ten and how many more?" (5.)
- Give the green strip (20 cm, square) and the blue strip (15 cm, triangle) from lesson 5, not lined up. "Which is longer? Show me how you know." (Green; the child lines up the ends, or puts them side by side from the same starting point.) Then the star and moon tubs: "Which is heavier?" (The sand tub; check which label it has before you start.)

Pointing, gesture, sign or a communication device all count as answers. Name strips by colour and shape ("the green square strip") for children with colour-vision differences. Record yes, with help or not yet for each part.

Sources

- Common Core State Standards, Kindergarten Operations and Algebraic Thinking (read to check what is expected in kindergarten, not to claim alignment): https://www.thecorestandards.org/Math/Content/K/OA/
- K.OA.A.1 represent addition and subtraction with objects, fingers, mental images, drawings, sounds (for example claps), acting out situations, verbal explanations, expressions, or equations; K.OA.A.2 solve addition and subtraction word problems, and add and subtract within 10; K.OA.A.5 fluently add and subtract within 5.
- Common Core, Kindergarten Number and Operations in Base Ten: https://www.thecorestandards.org/Math/Content/K/NBT/
- K.NBT.A.1 compose and decompose numbers from 11 to 19 into ten ones and some further ones, and record them by a drawing or equation (such as 18 = 10 + 8).
- Common Core, Kindergarten Measurement and Data: https://www.thecorestandards.org/Math/Content/K/MD/
- K.MD.A.1 describe measurable attributes of objects, such as length or weight; K.MD.A.2 directly compare two objects with a measurable attribute in common and describe the difference (its example compares the heights of two children).

Known gaps and cautions

- Story problems here are mostly "add to" and "take from" with the result unknown. Problems where the start or change is unknown ("Some birds were there, 2 more came, now there are 5; how many were there at first?") are harder and are only a stretch here.
- Number sentences (+, - and =) are shown by the teacher and read together; writing them is optional. Add more written work if your setting expects it.
- English teen-number words hide the ten ("fourteen" says the 4 first, and "eleven" and "twelve" do not sound like "ten and 1" or "ten and 2"). Many other languages say teen numbers more regularly. Welcome counting in home languages.
- Measuring with units (counting how many cubes long) is not taught here; this unit only compares two objects directly.
- Pouring to compare how much containers hold is only a stretch activity (lesson 5), because it needs water or sand play set up.
- The Common Core example compares two children's heights. This unit compares objects instead, so that no child's body is compared in front of the class.
- The lesson order and timings are my choice, not taken from a source.
- Peer check: Gemini 4 Argon did a bounded text, answer-key and timing check on 6 November 2026 at 0f3aeb0 (recorded in PEERCHECKS.md); fixes are in 4abff39, which Argon also diff-checked the same day.
- Peer check: GPT-6.1 Terra did a bounded text and activity check on 9 November 2026 at 5c3db56 (recorded in PEERCHECKS.md); Terra's three lesson 5 points are fixed in aeb39b7, which Terra also diff-checked the same day (see PEERCHECKS.md); Terra's one residual note (check beforehand that the sand tub is heavier; adult-set-up stable balance) is fixed in c974026, and a follow-up (use a scale if the box does not fit the balance) in 131c037.

Lesson 1: Putting together

Unit overview - Next: Lesson 2

About 30 minutes.

Goal

Children act out and solve short story problems where more are added or two groups are put together, with totals up to 10.

Materials

- About 10 small toys per group of four (toy animals or cars).
- A blue cloth or paper "pond" for the carpet.
- Counters in two colours (10 per child, at least 5 red and 3 blue) and whiteboards or paper.

Steps

✎ Opening (5 min)

Tell a story and let children act it out. "3 ducks are on the pond." Three children stand on the blue cloth. "2 more ducks come." Two more join. "How many ducks now?" Count together: 5. Say: "3 and 2 more make 5."

✎ Main activity (20 min)

1. **Toy stories (8 min).** In groups, an adult tells stories with the toys: "4 cows are in the field. 2 more come. How many now?" (6.) "5 cars are in the car park. 3 more drive in." (8.) Children move the toys and count.
2. **Two groups together (7 min).** "Here are 4 red counters and 3 blue counters. How many counters altogether?" (7.) Children push the two groups together and count. Some will count on from 4 ("5, 6, 7"); praise both ways.
3. **Clap it, draw it (5 min).** "Clap 2 times. Now clap 3 more. How many claps?" (5.) Then children draw one story: dots or simple shapes are fine. The teacher writes the number sentence under one drawing, for example 4 + 2 = 6, and reads it: "4 and 2 make 6."

✓ Discussion questions (with answers)

- How did you find out how many there were altogether? Counting all, counting on from the first group, or fingers are all good answers.
- 4 cows and 2 more. Is the answer more or less than 4? More; we added some.
- Does it matter if we put the red counters first or the blue ones? No, 4 and 3 make 7, and 3 and 4 make 7.

✓ Closing check (with answers)

"6 bears are at the picnic. 2 more come. Show me how many with your fingers." (8.) Then: "I clap 1 time, then 4 more times. How many claps?" (5.)

Vocabulary

add, more, altogether, and, make, put together

Differentiation

- **Support:** totals up to 5; the adult moves the toys while the child counts.
- **Stretch:** a "start unknown" story: "Some birds sat on a fence. 2 more came. Now there are 5. How many were there at first?" (3.) Or reading the number sentence on the board.
- Children who do not want to act on the carpet can move toys instead. Pointing to numeral cards counts as an answer.
- Any child can show answers with counters, numeral cards or taps on the table instead of fingers or claps.

⚠ Safety

Use toys and counters too large to swallow. Keep acting-out moves to walking on the carpet.

Teacher notes and misconceptions

- Some children recount everything from 1 each time. That is fine at this stage; counting on is a later habit, so show it but do not insist.
- Children sometimes count the first group twice when they push groups together. Ask them to touch each one once, as in term 1.

Lesson 2: Taking away

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 30 minutes.

Goal

Children act out and solve story problems where some are taken away, or a group is split into two parts, starting from up to 10.

Materials

- About 10 small toys per group of four.
- Counters in two colours (10 per child, at least 5 red and 3 blue) and whiteboards or paper.
- 5 paper or felt apples and a picture of a tree (optional, for the opening).

Steps

✎ Opening (5 min)

"5 apples are on the tree." Put 5 apples on the tree picture. "The wind blows and 2 fall down." Take 2 away. "How many are still on the tree?" (3.) Say: "5 take away 2 is 3."

✎ Main activity (20 min)

1. **Toy stories (8 min).** "9 birds are on a wall. 4 fly away. How many are still on the wall?" (5.) "7 cars are in the car park. 2 drive away." (5.) "6 frogs are on a log. 6 jump in the water." (0; no frogs are left.) Children move the toys away and count what is left.
2. **Split it (7 min).** "Here are 6 toy animals. 2 are cats and the rest are dogs. How many dogs?" (4.) Children sort the toys into two groups and count each group. Do the same with 8 counters: 5 red and the rest blue. (3 blue.)
3. **Draw and cross out (5 min).** Children draw 7 circles and cross out 3. "How many are not crossed out?" (4.) The teacher writes 7 - 3 = 4 and reads it: "7 take away 3 is 4."

✓ Discussion questions (with answers)

- When we take some away, do we have more or fewer than before? Fewer.
- How did you find how many were left? Counting what was left, counting back, or fingers are all good answers.
- 6 frogs, and 6 jump away. How many are left? None, which is 0.

✓ Closing check (with answers)

"8 cars are in the car park. 3 drive away. Show me with your fingers how many are left." (5.)
Then: "10 balloons, 1 pops. How many now?" (9.)

Vocabulary

take away, left, fewer, none, zero, split

Differentiation

- **Support:** start from 5 or fewer; the child takes the toys away one at a time while counting.
- **Stretch:** "I had some sweets and ate 2. Now I have 3. How many did I have at first?" (5.) Or reading the number sentence on the board.
- Children may point to numeral cards or sign answers.
- Any child can show answers with counters, numeral cards or taps on the table instead of fingers or claps.

⚠ Safety

Use toys and counters too large to swallow.

Teacher notes and misconceptions

- Some children count the toys that went away instead of the ones left. Ask "Which ones are still here?" and point to them.
- Stories with food or animals should avoid upsetting events. Wind, flying away and driving away work well.

Lesson 3: Quick facts within 5

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 30 minutes.

Goal

Children add and take away within 5 quickly, using fingers, quick looks and the ways to make 5 from term 2.

Materials

- 5 counters and a paper cup per pair.
- Dot cards showing 0 to 5 dots (two sets per pair, mixed together, so that doubles such as 2 and 2 can come up).
- Numeral cards 0 to 5 for children who prefer to point.

Steps

✎ Opening (5 min)

Fingers warm-up: "Show me 3 fingers. Now 1 more. How many?" (4.) "Show me 5. Put 2 down. How many are up?" (3.) Go quickly; it is fine to look at your fingers.

✎ Main activity (20 min)

1. **Hide and show (8 min).** In pairs, one child puts 5 counters on the table, then hides some under the cup while the partner closes their eyes. The partner looks at what is left: "3 are showing, so 2 are hiding." Lift the cup to check. Swap roles. Then play with 4 counters.
2. **Dot card add (7 min).** Each child turns over two dot cards (for example 2 dots and 3 dots) and says the total (5). If the total is more than 5, put the cards back and try again.
3. **Quick quiz (5 min).** The teacher calls out facts and children show the answer with fingers: 2 and 2 (4), 1 and 3 (4), 4 and 1 (5), 5 take away 2 (3), 4 take away 1 (3), 5 take away 5 (0). The teacher writes two of them as number sentences, for example 2 + 2 = 4 and 5 - 2 = 3.

✓ Discussion questions (with answers)

- 3 counters are showing out of 5. How did you know 2 were hiding? We know 3 and 2 make 5; or we counted on from 3 to 5.
- Is 1 and 4 the same as 4 and 1? Yes, both make 5.
- What is 4 take away 0? 4; nothing was taken.

✓ Closing check (with answers)

Children show fingers: "3 and 1?" (4.) "5 take away 3?" (2.) "2 and 3?" (5.)

Vocabulary

and, make, take away, the same as, hiding, showing

Differentiation

- **Support:** play hide and show with 3 counters; use the numeral cards.
- **Stretch:** hide and show with 10 counters (links to make 10 in term 2).
- "Quick" means a few seconds, not a race. Do not time children or make the quiz a competition.
- Any child can show answers with counters, numeral cards or taps on the table instead of fingers or claps.
- Instead of closing their eyes, the partner can turn away, or the child can hide counters behind a card screen.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Children who already know a fact "by heart" still benefit from showing it with fingers or counters now and then.
- Short, frequent practice (a few facts a day in the rest of the term) helps more than one long session.

Lesson 4: Teen numbers: ten and some more

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 30 minutes.

Goal

Children build numbers from 11 to 19 as one full ten-frame and some more ones, and say them as "ten and ...".

Materials

- Two ten-frames and 20 counters per pair.
- A large pair of ten-frames for the carpet.
- Numeral cards 10 to 19.
- Optional: 20 lolly sticks or straws and a rubber band per pair, to make a bundle of ten.

Steps

✎ Opening (5 min)

Fill the big ten-frame, counting together to 10. "It is full. That is ten." Put 3 more counters on the second frame. "Ten and 3 more. Let's count on: 11, 12, 13. Ten and 3 is 13." Show the numeral card 13: "The 1 is for the full ten, and the 3 is for the 3 more." Write 13 = 10 + 3 on the board.

✎ Main activity (20 min)

1. **Build it (8 min).** The adult shows a numeral card from 11 to 19. Pairs fill the first frame to 10 and put the rest on the second frame. "What is 16?" (Ten and 6.) "What is 19?" (Ten and 9.) "What is 11?" (Ten and 1.)
2. **What number is it? (7 min).** The adult builds a full frame and some more. Children say the number: ten and 7 (17), ten and 4 (14), ten and 8 (18). Children who can, find the numeral card.
3. **Bundle of ten (5 min, optional).** Pairs count 10 sticks, put a rubber band round them, and add some loose sticks. "One bundle of ten and 5 loose sticks. How many?" (15.)

✓ Discussion questions (with answers)

- How do we know the first frame has 10 without counting? Every box is full, and a full frame is always 10.
- In the number 14, what does the 4 tell us? 4 more ones after the ten.
- Which is more: ten and 2, or ten and 8? Ten and 8 (18 is more than 12).

✓ Closing check (with answers)

Show a full frame and 6 more. "How many?" (16.) Then: "Show me ten and 2 with your counters." (A full frame and 2 more; 12.)

Vocabulary

ten, full, ones, ten and ..., teen numbers

Differentiation

- **Support:** start with 11, 12 and 13; count every counter from 1 and then say "ten and ...".
- **Stretch:** "What is 10 + 8?" without counters (18); or "18 is ten and how many?" (8.)
- Children who find fine motor work hard can use larger counters, or point while a partner places them.

⚠ Safety

Use counters too large to swallow. Rubber bands: an adult puts them on, and they are collected at the end.

Teacher notes and misconceptions

- In English, "fourteen" says the 4 first but we write 1 then 4, and "eleven" and "twelve" don't sound like ten and 1 or ten and 2. Expect reversals such as writing 41 for fourteen; showing ten and 4 on frames helps.
- Always fill the first frame completely before using the second, so the ten is easy to see.
- Some children think the 1 in 14 means one counter. The full ten-frame is what that 1 stands for; this is the start of place value, not the whole idea.

Lesson 5: Longer, shorter, heavier, lighter

Unit overview - Previous: Lesson 4

About 30 minutes.

Goal

Children compare two objects by length and by weight, by lining them up or holding them, and use the words longer, shorter, heavier and lighter.

Materials

Per group of four:

- Four paper strips in different colours and clearly different lengths: red 25 cm, blue 15 cm, green 20 cm, yellow 10 cm. Also draw a shape on each so colour is not the only clue: circle on yellow, triangle on blue, square on green, heart on red.
- A pencil, a crayon and a shoe.
- Masking tape stuck down along one edge of the table as a start line.
- Two lidded tubs of the same size: one full of sand (labelled with a star) and one full of cotton wool (labelled with a moon). Fill and close them before the lesson.

For the class:

- A simple bucket balance. If you do not have one, make one: hang a coat hanger from a hook or a broom handle held across two chairs, and tie a plastic bag or cup to each end at the same distance from the middle. Check it hangs level when empty. An adult sets it up so it is stable and cannot fall on children.
- A large empty cardboard box and a small tub of sand, for the opening.

Steps

✎ Opening (5 min)

Hold up the green strip (20 cm) and the blue strip (15 cm), with the green one held lower so the tops are uneven. "Which is longer?" Take guesses. Then put both on the floor with their ends on the same line. "Now we can see: the green strip is longer. To compare, we start them at the same place."

✎ Main activity (20 min)

1. **Start at the line (7 min).** Groups put two strips at a time on the start line and say which is longer and which is shorter. Then they put all four in order, shortest to longest (yellow, blue, green, red). Then they compare the pencil, the crayon and the shoe the same way, and say what they found (answers depend on the objects; any comparison made from the start line is right).

2. **Hold and feel (7 min).** Each child holds the star tub in one hand and the moon tub in the other. "Which is heavier? Which is lighter?" (The star tub, with sand, is heavier.) "They are the same size. Can you tell which is heavier just by looking?" (No; we had to hold them.)
3. **Balance (6 min, whole class).** Show the big empty box and the small tub of sand. "Which is heavier?" Take guesses, then put them in the balance. The side that goes down is heavier (usually the small sand tub). If the box is too big for your balance, children take turns holding the box in one hand and the sand tub in the other instead. Before the lesson, check that your sand tub really is heavier than the empty box (on the balance if the box fits, otherwise on a kitchen scale or a bigger balance); if not, add more sand or use a lighter box. "The bigger one is not always the heavier one."

✓ Discussion questions (with answers)

- Why do we put the strips on the same start line? So we compare the lengths fairly; if one starts further along, it can look longer when it is not.
- How can we find out which tub is heavier? Hold one in each hand, or use a balance.
- On a balance, which side is heavier? The side that goes down.

✓ Closing check (with answers)

Hold up the red strip (25 cm) and the yellow strip (10 cm) and line them up on the start line in front of the children. "Which is shorter?" (Yellow.) Then a child holds the star and moon tubs. "Which is lighter?" (The moon tub, with cotton wool.)

Vocabulary

longer, shorter, heavier, lighter, the same, start line, balance, compare

Differentiation

- **Support:** compare just two strips with a large difference (red and yellow).
- **Stretch:** pouring play: which of two containers holds more? Fill one with sand or water and pour it into the other. If it overflows, the first held more. If it does not overflow, that does not yet show which holds more: fill the second container and pour it back into the first. If that overflows, the second holds more; if neither overflows and each one fills the other, they hold about the same. A tall thin bottle and a short wide bowl often surprise children.
- Children who cannot hold the tubs can have each tub rested on their lap or palm in turn (a helper holds it steady), or watch the balance and say which side went down. (Pushing tubs along a table is not a fair way to compare weight: the tubs may slide differently.) Children with low vision can compare strips by touch, with ends lined up against the tape.
- For children with colour-vision differences, name strips by colour and shape ("the green square strip").

⚠ Safety

Keep the tubs light enough for a child to hold easily (a small yogurt-pot size of sand is enough) and check the lids are taped on. For pouring play, put down towels and wipe up spills at once so nobody slips. Keep sand away from faces.

Teacher notes and misconceptions

- Children often judge length by where an object ends, not by where it starts. The start line is the key habit here.
- Many children think bigger always means heavier. The box and sand tub show that this is not always true.
- Compare objects, not children. Comparing children's heights in front of the class can upset some children.

Grade 1 Mathematics: Adding and Taking Away Within 20 (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 9 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

Back to the grade 1 year plan - Term 1. Builds on kindergarten term 3: Adding, taking away, teen numbers and comparing

Big idea

Adding and taking away are connected. A story problem can be about adding to, taking from, putting two parts together, splitting a whole into parts, or comparing two groups, and the unknown number can be at the end, in the middle or at the start. We do not have to count everything from 1: we can count on, make a ten, use doubles, or turn a take-away into "what do I add?". The equal sign means "is the same amount as", not "the answer comes next".

Outcomes

By the end of the unit most children can:

1. Solve story problems within 20 of all five kinds (adding to, taking from, putting together, taking apart, comparing), with the unknown at the end or in the middle, using objects, drawings or a number sentence with a box for the unknown.
2. Add by counting on from the larger number (for example 3 + 12: start at 12, say "13, 14, 15") and take away small numbers by counting back.
3. Add within 20 by making a ten (8 + 6 = 8 + 2 + 4 = 10 + 4 = 14) and add three numbers by finding two that make ten.
4. Use doubles and near doubles (6 + 7 = 6 + 6 + 1 = 13), and solve a take-away by thinking of the missing part (10 - 8: "8 and what make 10?").
5. Say whether a number sentence such as 4 + 1 = 5 + 2 is true or false, and find the missing number in sentences such as 8 + □ = 11.
6. Add and take away within 10 quickly (within a few seconds, with or without fingers).

Prior knowledge

The kindergarten units: counting to 20, ways to make 5 and 10 on a ten-frame, teen numbers as "ten and some more", and simple adding and taking-away stories within 10. Children still building these can work within 10 in every lesson.

Lessons

Each lesson is about 40 minutes: a short whole-class start, paired or group work with objects, and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so each strategy should come back in short daily practice (5 minutes of quick facts or one story problem).

1. Story problems: adding, taking away, parts and wholes
2. Counting on and counting back
3. Making ten
4. Doubles, near doubles and fact families
5. Comparing and the equal sign

Materials (whole class unless noted)

- Linking cubes too large to swallow, in two colours: 20 per child (lessons 1 to 5). If colours are hard to tell apart, use two kinds of object instead (cubes and large buttons), so colour is not the only clue.
- Two ten-frames per child (lesson 3); one large ten-frame pair for the board.
- A 0 to 20 number line and a counter per pair (lesson 2), with numbers large enough to read and, if possible, raised marks (glue dots) at 0, 5, 10, 15 and 20. Optional: masking tape for a 0 to 20 track on the floor.
- A part-whole mat per pair (a sheet with one large box on top for the whole and two boxes below for the parts) for lessons 1 and 4.
- Digit cards 0 to 20 per pair, used for pointing to answers in every lesson.
- For lesson 3: a set of cards 1 to 9 per pair (for the pairs-to-ten game).
- For lesson 4, per group: two clean 12-hole egg boxes (or a 2 by 6 grid drawn on card); per pair: a set of fact-family triangle cards (see lesson 4).
- For lesson 5, per pair: true-or-false cards and missing-number cards (see lesson 5), support cards with one number on each side (such as 6 = 6 and 5 = 4), and two trays marked by touch (a rough sticker for true, a smooth one for false). Optional: a toy or home-made balance.
- Whiteboards and pens, or paper and pencils, for drawings and number sentences.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week:

- "6 frogs are on a log. Some more hop on. Now there are 10. How many hopped on?" (4.) The child may use cubes, a drawing or 6 + □ = 10.
- "What is 8 + 5? Show me how you worked it out." (13. Making ten is 8 + 2 + 3 = 10 + 3; counting on from 8 is also fine.)
- "What is 6 + 7?" (13; a near double, 6 + 6 + 1, or any working strategy.)
- "What is 12 - 4?" (8; counting back, or "4 and what make 12?")
- Show the card 4 + 1 = 5 + 2. "Is this true or false?" (False: 4 + 1 is 5 and 5 + 2 is 7.)
- Quick facts within 10: 6 + 3 (9), 9 - 4 (5), 7 + 2 (9). An answer within a few seconds, with or without fingers, counts as quick.

Pointing to digit cards or the number line, sign, writing, or a communication device all count as answers. Record yes, with help or not yet for each part.

Sources

- Common Core State Standards, Grade 1 Operations and Algebraic Thinking (read to check what is usually expected in grade 1, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/OA/
- 1.OA.A.1 word problems within 20 of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using objects, drawings and equations with a symbol for the unknown; 1.OA.A.2 adding three whole numbers with a sum up to 20; 1.OA.B.3 order and grouping as strategies (its examples are 8 + 3 = 3 + 8 and 2 + 6 + 4 = 2 + 10 = 12), with no need for the formal names; 1.OA.B.4 subtraction as an unknown-addend problem (10 - 8); 1.OA.C.5 relating counting to adding and taking away; 1.OA.C.6 adding and taking away within 20, fluency within 10, with strategies such as counting on, making ten (8 + 6 = 8 + 2 + 4), decomposing to a ten (13 - 4 = 13 - 3 - 1), and known sums (6 + 7 = 6 + 6 + 1); 1.OA.D.7 the meaning of the equal sign and true or false equations (its examples include 7 = 8 - 1 and 4 + 1 = 5 + 2); 1.OA.D.8 finding the unknown in equations such as 8 + ? = 11 and 5 = _ - 3.
- Common Core, Grade 1 introduction (four critical areas, the first being addition, subtraction and strategies within 20): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/introduction/

Known gaps and cautions

- Problems with the unknown at the start ("Some birds were on a wire, 3 more came, now there are 9") and comparing problems where the larger amount is unknown are the hardest kinds. As story problems they appear in lessons 1 and 5 only as stretch tasks; plan to return to them in term 2. Bare missing-number equations with the unknown first (5 = □ - 3, □ + 4 = 13) are core in lesson 5, step 3, because the box is easier to see than in a story.
- Fluency within 20 is not expected by the end of this unit; only within 10.
- The number line is used for counting on and back from small jumps only. Using it for bigger jumps comes later.
- The names in story problems are made up. Swap in names and objects that suit your class, but avoid using real children's belongings or families.
- Peer check: Gemini 4 Argon did a bounded text, timing and answer-key check at e844112 (not an educator or accessibility review). Keys and timings were correct. Its notes (materials lists out of step with lessons 2, 4 and 5; the start-unknown note did not match lesson 5's core missing-number cards; the equal-sign answer called 8 - 1 a sum) were fixed in the follow-up commit 8373393 (see PEERCHECKS.md), which Argon also diff-checked the same day.
- The lesson order and timings are my choice, not taken from a source.

Lesson 1: Story problems: adding, taking away, parts and wholes

Unit overview - Next: Lesson 2

About 40 minutes.

Goal

Children solve story problems within 20 about adding to, taking from, putting together and taking apart, with the unknown at the end or in the middle, and show them with cubes, drawings and number sentences.

Materials

- 20 linking cubes per child, in two colours (or two kinds of object).
- A part-whole mat per pair.
- Digit cards 0 to 20 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Remind children of kindergarten stories. "5 ducks are on the pond. 3 more come. How many now?" (8.) Write 5 + 3 = 8 and read it: "5 plus 3 equals 8." Then write 5 + 3 = □ and say: "The box is the number we don't know yet."

✎ Main activity (30 min)

1. **Adding to and taking from (10 min).** Tell each story twice; pairs use cubes or draw, then point to or write the answer.
    - "9 birds sit on a wire. 4 fly away. How many are left?" (5; 9 - 4 = 5.)
    - "7 children are on the bus. 6 more get on. How many now?" (13; 7 + 6 = 13.)
    - "12 apples are in a bowl. 5 are eaten. How many are left?" (7; 12 - 5 = 7.)
2. **Putting together and taking apart (10 min).** Show the part-whole mat: the whole goes in the top box, the two parts below.
    - "8 red flowers and 7 yellow flowers are in a pot. How many flowers?" (15; parts 8 and 7, whole 15.)
    - "There are 14 socks. 6 are spotty and the rest are striped. How many are striped?" (8; whole 14, one part 6, so the other part is 8.)
    - "11 cubes: split them into two towers any way you like. Write your two parts." (Any pair that makes 11, such as 5 and 6, 3 and 8, or 0 and 11.)
3. **The missing middle (10 min).** The change is unknown.
    - "6 frogs are on a log. Some more hop on. Now there are 10. How many hopped on?" (4; 6 + □ = 10.)

- "Ana had 11 cards. She gave some away. Now she has 7. How many did she give away?" (4; 11 - □ = 7.) Let children act it out with cubes: start with 6, add one at a time and count how many were added until there are 10.

✓ Discussion questions (with answers)

- In the bird story, did the number get bigger or smaller? Why? Smaller, because some birds flew away.
- In the sock story, nothing came or went. What happened instead? The socks were split into two groups; we knew the whole and one part.
- In the frog story, why is the answer not 10? 10 is how many there are at the end. The question asks how many hopped on, which is 4.

✓ Closing check (5 min, with answers)

"There are 13 buttons. 5 are red and the rest are blue. How many are blue?" Children draw, use cubes or point to a card. (8.) Then: "Write or point to a number sentence for it." (5 + 8 = 13, 13 - 5 = 8 or 5 + □ = 13 are all good.)

Vocabulary

plus, minus, equals, add, take away, altogether, left, part, whole, number sentence

Differentiation

- **Support:** totals up to 10; an adult reads the story again while the child moves cubes one at a time.
- **Stretch:** start-unknown stories: "Some birds were on a wire. 3 more came. Now there are 9. How many were there at first?" (6; □ + 3 = 9.)
- Any child can answer by pointing to a digit card, showing cubes, signing or using a communication device. Stories can be shown with pictures as well as told.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Many children add any two numbers they hear. In the frog story they may say 16 (6 + 10). Ask them to act it out and check the story still makes sense.
- Keep the story language varied ("left", "now", "at first", "the rest") so children listen to the story rather than to key words.
- Several number sentences can fit one story; accept any that match it.

Lesson 2: Counting on and counting back

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 40 minutes.

Goal

Children add by counting on from the larger number, and take away 1, 2 or 3 by counting back, using a 0 to 20 number line.

Materials

- A 0 to 20 number line and a counter per pair.
- 20 linking cubes per child.
- Digit cards 0 to 20 per pair.
- Optional: a 0 to 20 track taped on the floor.

Steps

✎ Opening (5 min)

Count together from 0 to 20, then start from other numbers: "Start at 8 and count on 4: 9, 10, 11, 12." Children may hold up a finger for each number they say. If you have a floor track, volunteers can step along it while others count; children who prefer can move a counter on their table line instead.

✎ Main activity (30 min)

1. **Count on (10 min).** "9 + 3. Put your counter on 9. Don't count the 9 - we already have it. Jump 3: 10, 11, 12." (12.) Pairs try:
    - 8 + 3 (11)
    - 14 + 2 (16)
    - 15 + 4 (19) Check one with cubes: a tower of 9 and 3 more cubes is 12.
2. **Start with the bigger number (10 min).** "2 + 9. Is it quicker to start at 2 or at 9?" Build a tower of 2 and a tower of 9, join them, and turn the train round: it is the same train, 11 cubes. So 2 + 9 = 9 + 2 = 11. Pairs try, starting from the bigger number:
    - 3 + 12 (15)
    - 1 + 16 (17)
    - 4 + 13 (17)
3. **Count back (10 min).** "13 - 2. Put your counter on 13. Jump back 2: 12, 11." (11.) Pairs try:
    - 17 - 3 (14)
    - 20 - 1 (19)
    - 16 - 2 (14) Then check one with cubes: 17 cubes, take 3 away, count what is left (14).

✓ Discussion questions (with answers)

- When we count on from 9 for 9 + 3, why don't we say "9" again? The 9 is already there; we count only the 3 new jumps.
- Is 3 + 12 the same as 12 + 3? Yes. Both are 15; the same cubes in a different order.
- Would you count back to work out 15 - 9? It is slow and easy to lose track. Another way (thinking "9 and what make 15?") comes in lesson 4.

✓ Closing check (5 min, with answers)

"4 + 11. Show me where you start and where you land." (Start at 11; land on 15.) "16 - 3." (13.)

Vocabulary

count on, count back, jump, number line, start, land, the same as

Differentiation

- **Support:** totals up to 10; count on with cubes before using the line.
- **Stretch:** "I landed on 18 after jumping 3. Where did I start?" (15.)
- Number lines with large print and glue-dot marks at 0, 5, 10, 15 and 20 help children with low vision; they can also count cubes by touch. Children can give answers by pointing to the line, to digit cards, or by signing.

⚠ Safety

Floor tracks: walking only, with space between children. Keep the track away from doors and table corners.

Teacher notes and misconceptions

- The most common slip is counting the starting number as a jump: for 9 + 3 saying "9, 10, 11" and answering 11. Ask children to say the numbers they land on, not the number they start on.
- Counting back by more than 3 is error-prone; for bigger take-aways, use cubes or the strategies in lessons 3 and 4.

Lesson 3: Making ten

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 40 minutes.

Goal

Children know the pairs that make 10, use them to add within 20 by making a ten, and add three numbers by finding two that make ten.

Materials

- Two ten-frames and 20 counters or cubes per child, in two colours (or two kinds of object).
- Cards 1 to 9 per pair.
- A large pair of ten-frames for the board.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Quick pairs to ten. Show 7 on the big ten-frame. "How many more make 10?" (3.) Repeat with 4 (6), 9 (1), 5 (5) and 2 (8). Children answer with fingers, cubes or digit cards.

✎ Main activity (30 min)

1. **Pairs-to-ten game (8 min).** Pairs shuffle cards 1 to 9 and place them face down. One child turns a card; the other says, shows or points to the number that makes 10 with it. Swap after each card.
2. **Make a ten to add (12 min).** On the board: 8 + 6. Put 8 counters on the first ten-frame and 6 on the second. "The first frame needs 2 more to be full. Move 2 across." Now there are 10 and 4: 14. Write 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14. Pairs try with their frames:
    - 9 + 5 (14: move 1, so 10 + 4)
    - 7 + 5 (12: move 3, so 10 + 2)
    - 8 + 7 (15: move 2, so 10 + 5)
    - 9 + 8 (17: move 1, so 10 + 7)
3. **Three numbers (10 min).** "2 + 6 + 4. Can you see two numbers that make ten?" (6 and 4.) So 2 + 10 = 12. Pairs try:
    - 7 + 5 + 3 (15: 7 + 3 = 10, then 5 more)
    - 1 + 8 + 9 (18: 1 + 9 = 10, then 8 more)
    - "Bo has 3 red pencils, 5 blue pencils and 7 green pencils. How many pencils?" (15: 3 + 7 = 10, then 5 more.)

✓ Discussion questions (with answers)

- Why is 10 + 4 easier than 8 + 6? Ten and some more is a teen number we already know: ten and 4 is 14.
- For 9 + 5, why do we move just 1? 9 needs only 1 more to make 10.
- Does it matter which order we add three numbers in? No; the total is the same, so we can pick the order that makes a ten.

✓ Closing check (5 min, with answers)

"9 + 4. Show me on your ten-frames." (13: move 1, 10 + 3.) "5 + 3 + 5." (13: 5 + 5 = 10, then 3 more.)

Vocabulary

make ten, pair, full, ten-frame, ten and some more

Differentiation

- **Support:** stay with pairs to ten and with 9 + a small number, where only 1 counter moves.
- **Stretch:** take away through ten: 13 - 4 = 13 - 3 - 1 = 10 - 1 = 9. Use the ten-frames: take away the 3 loose counters first, then 1 from the full frame.
- Ten-frames can be made tactile (counters in an egg box or a 2 by 5 grid of cups) for children with low vision. Children can answer by pointing to digit cards or moving counters instead of speaking.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Some children move the wrong number of counters, or move counters from the first frame to the second. Ask "Which frame is nearly full?" first.
- Writing 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14 is a teacher record. Children do not need to write chains like this yet, and it should not be used to suggest that "=" means "and then".

Lesson 4: Doubles, near doubles and fact families

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 40 minutes.

Goal

Children use doubles and near doubles to add, and solve a take-away by thinking of the missing part ("8 and what make 10?").

Materials

- Two empty 12-hole egg boxes (or a 2 by 6 grid on card) and 12 cubes per group. One box shows doubles up to 6 + 6; the second box lets the group show doubles up to 10 + 10, one row of 10 holes across the two boxes.
- 20 linking cubes per child.
- A part-whole mat per pair.
- Fact-family triangle cards per pair: triangles with three numbers at the corners, for example 8, 4, 12; 3, 7, 10; 6, 9, 15. The biggest number (the whole) is at the top corner.
- Digit cards 0 to 20 per pair.
- Whiteboards and pens, or paper and pencils, for recording number sentences.

Steps

✎ Opening (5 min)

Doubles. Fill one row of an egg box with 6 cubes and the other row with 6. "6 and 6 make?" (12.) Children build cube towers in pairs of the same height: 5 and 5 (10), 7 and 7 (14), 8 and 8 (16), 9 and 9 (18), 10 and 10 (20).

✎ Main activity (30 min)

1. **Doubles practice (8 min).** In pairs, one child builds a tower of up to 10 cubes; the other builds the same and says the double. Record: 7 + 7 = 14.
2. **Near doubles (10 min).** "6 + 7. 7 is one more than 6, so it is double 6 and 1 more." 6 + 6 + 1 = 12 + 1 = 13. Pairs check with towers, then try:
    - 7 + 8 (15: 7 + 7 + 1)
    - 8 + 9 (17: 8 + 8 + 1)
    - 5 + 6 (11: 5 + 5 + 1)
3. **Fact families and "what do I add?" (12 min).** Take the triangle 8, 4,
4. With cubes on the part-whole mat: whole 12, parts 8 and 4. Together find four number sentences: 8 + 4 = 12, 4 + 8 = 12, 12 - 8 = 4, 12 - 4 = 8. Then: "10 - 8. Think: 8 and what make 10?" (2.) Pairs try:
    - 15 - 7 (think 7 + □ = 15; 8)
    - 13 - 9 (think 9 + □ = 13; 4)

- Write the four sentences for the triangle 3, 7, 10 (3 + 7 = 10, 7 + 3 = 10, 10 - 3 = 7, 10 - 7 = 3).

✓ Discussion questions (with answers)

- How does knowing 6 + 6 help with 6 + 7? 6 + 7 is one more than 6 + 6, so it is 13.
- If you know 8 + 4 = 12, what else do you know? 4 + 8 = 12, 12 - 8 = 4 and 12 - 4 = 8.
- For 13 - 9, why is thinking "9 and what make 13?" easier than counting back 9? Counting back 9 is long and easy to lose track of; 9 needs 1 to make 10 and 3 more to make 13, so 4.

✓ Closing check (5 min, with answers)

"7 + 7." (14.) "7 + 8." (15.) "14 - 7." (7, because 7 + 7 = 14.)

Vocabulary

double, near double, one more, fact family, whole, part

Differentiation

- **Support:** doubles up to 5 + 5 with fingers or cubes, and fact families for totals up to 10.
- **Stretch:** near doubles the other way: 8 + 7 = 8 + 8 - 1 = 15. Or make their own triangle card and write all four sentences.
- Egg boxes and cube towers can be checked by touch. Children can show answers with cubes, digit cards or sign.

⚠ Safety

Use clean egg boxes (washed or never used for eggs) and cubes too large to swallow.

Teacher notes and misconceptions

- Some children think 12 - 8 and 8 - 12 are the same, because 8 + 4 and 4 + 8 are. Show with cubes that we cannot take 12 away from 8 cubes; in a take-away, the whole comes first.
- Near doubles only help once doubles are quick. Keep short daily doubles practice going.

Lesson 5: Comparing and the equal sign

Unit overview - Previous: Lesson 4

About 40 minutes.

Goal

Children solve "how many more?" and "how many fewer?" problems, decide whether number sentences are true or false, and find missing numbers.

Materials

- 20 linking cubes per child, in two colours (or two kinds of object).
- True-or-false cards, one set per pair: 6 = 6; 7 = 8 - 1; 5 + 2 = 2 + 5; 4 + 1 = 5 + 2; 10 = 6 + 4; 9 + 1 = 10 + 1.
- Missing-number cards: 8 + □ = 11; 5 = □ - 3; 6 + 6 = □; □ + 4 = 13; 15 - □ = 9.
- Optional: a toy or home-made balance and equal-sized cubes.
- Digit cards 0 to 20 per pair; whiteboards and pens.
- Support cards with one number on each side (6 = 6, 5 = 4), and two trays marked by touch (a rough sticker for true, a smooth one for false), for children who need them (see Differentiation).

Steps

✎ Opening (5 min)

Build a tower of 9 cubes and a tower of 6, side by side. "How many more cubes does the tall tower have?" Children see 3 extra at the top. "9 is 3 more than 6. 6 is 3 fewer than 9."

✎ Main activity (30 min)

1. **Comparing stories (10 min).** Pairs build towers to match each story.
   - "Mia has 12 shells. Tom has 8. How many more shells does Mia have?" (4.)
   - "Bo has 5 stickers. Ana has 3 more than Bo. How many stickers does Ana have?" (8.)
   - "Sam has 11 marbles. Lee has 4 fewer than Sam. How many marbles does Lee have?" (7.)
2. **True or false? (10 min).** "The equal sign means both sides are the same amount." If you have a balance, show 3 + 2 cubes on one side and 5 on the other: it balances. Pairs sort the cards into true and false, using cubes to check:
   - 6 = 6 (true)
   - 7 = 8 - 1 (true)
   - 5 + 2 = 2 + 5 (true)
   - 4 + 1 = 5 + 2 (false: 5 on one side, 7 on the other)
   - 10 = 6 + 4 (true)

    - 9 + 1 = 10 + 1 (false: 10 and 11) Then: "Change one number to make 4 + 1 = 5 + 2 true." (For example 4 + 3 = 5 + 2, or 4 + 1 = 5 + 0.)
3. **Missing numbers (10 min).** Pairs find each box using cubes or a strategy from this unit:
    - 8 + □ = 11 (3)
    - 5 = □ - 3 (8)
    - 6 + 6 = □ (12)
    - □ + 4 = 13 (9)
    - 15 - □ = 9 (6)

✓ Discussion questions (with answers)

- Some people think 7 = 8 - 1 is wrong because the answer should come last. Is it wrong? No. Both sides are 7, so it is true; the equal sign can have the single number or the calculation on either side.
- In the sticker story, does "more" mean we add? Here yes: Ana has Bo's 5 and 3 more, 8. But in the shell story, "how many more" is found by comparing or taking away (12 - 8 = 4). Read the whole story, not one word.
- How did you find 5 = □ - 3? Think: what number, take away 3, leaves 5? 5 + 3 = 8.

✓ Closing check (5 min, with answers)

"Is 3 + 4 = 7 true? Is 7 = 3 + 4 true?" (Both true.) "9 + □ = 16." (7.)

Vocabulary

compare, more than, fewer than, how many more, equal sign, the same as, true, false, missing number

Differentiation

- **Support:** compare towers up to 10; true-or-false cards with only one number on each side first (6 = 6, 5 = 4).
- **Stretch:** comparing with the bigger amount unknown: "Lee has 7 marbles. That is 4 fewer than Sam. How many has Sam?" (11.) Or make up a true-or-false card for a partner.
- Children can sort cards by touch-marked trays (a rough sticker for true, a smooth one for false), point, sign or use a device. Read cards aloud for children who need it.

⚠ Safety

Use cubes too large to swallow. Keep a home-made balance low and steady on the table.

Teacher notes and misconceptions

- "=" read as "makes" leads children to call 7 = 8 - 1 false and to write answers such as 8 + □ = 11 with □ = 19 (adding all the numbers they see). Read "=" as "is the same as".

- Comparing problems are harder than adding to or taking from. Lining up two towers and looking at the extra part helps more than any rule.
- The names are made up; use names that do not belong to children in the class if comparisons might feel personal.

Grade 1 Mathematics: Tens and Ones to 100 (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 9 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

Back to the grade 1 year plan - Term 2. Builds on term 1: Adding and taking away within 20

Big idea

Ten ones can be grouped and treated as one thing, a "ten". A two-digit number tells us how many tens and how many ones: 47 is 4 tens and 7 ones. Because of this we can count big collections quickly, compare numbers by looking at the tens first, find 10 more or 10 less without counting in ones, and add by putting tens with tens and ones with ones.

Outcomes

By the end of the unit most children can:

1. Count to 120 starting from any number below 120, and read and write the numbers in that range.
2. Count a collection of up to about 100 objects by grouping it into tens, and say how many tens and ones there are (for example, 4 towers of ten and 7 loose cubes is 47).
3. Explain that 11 to 19 are a ten and some ones, and that 10, 20, 30 ... 90 are 1 to 9 tens and no ones.
4. Compare two two-digit numbers by looking at the tens and then, if the tens are the same, the ones, and record the result with >, = or <.
5. Find 10 more and 10 less than a two-digit number without counting in ones, and add and take away multiples of ten (30 + 40, 80 - 30).
6. Add a one-digit number or a multiple of ten to a two-digit number within 100, using cubes or drawings, including sums where the ones make a new ten (27 + 5 = 32).

Prior knowledge

The term 1 unit (adding and taking away within 20, making ten) and the kindergarten idea of teen numbers as "ten and some more". Children who are still building these can work with numbers to 50 in every lesson.

Lessons

Each lesson is about 40 minutes: a short whole-class start, paired work with cubes, and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so counting, 10 more and 10 less, and comparing should come back in short daily practice (5 minutes).

1. Grouping in tens and counting to 120
2. Tens and ones on a place-value mat

3. Comparing two-digit numbers
4. Ten more, ten less, and adding and taking tens
5. Adding within 100

Materials (whole class unless noted)

- Linking cubes too large to swallow, all one colour or mixed: 100 per pair (lessons 1 to 5). Children snap them into towers of ten.
- Optional for lesson 1: craft sticks, about 60 per pair, with strips of paper and tape to wrap bundles of ten (no rubber bands, which may contain latex).
- A tray or sheet of paper per pair to keep each collection together (lesson 1), and some collections to count: about 30 to 60 small objects per pair, too large to swallow (cubes, large buttons, counters).
- A place-value mat per pair (lessons 2, 3 and 5): a sheet split into a "tens" side on the left and a "ones" side on the right, with a strip of card glued down the middle as a ridge and a rough sticker on the tens side, so the two sides can be told apart by touch.
- Two sets of digit cards 0 to 9 per pair (lessons 2, 3 and 5), large print, with each digit outlined in glue or string (or with braille labels) so the cards can be read by touch.
- Symbol cards >, < and = per pair (lesson 3), with the symbol outlined in glue or string so it can be felt. Optional: a toy or home-made balance.
- A 1 to 100 chart (ten rows of ten, 1 at the top left) per pair and a large one for the board (lesson 4), with three counters per child. Large print; if possible, a raised dot on every number in the 10s column (10, 20 ... 100).
- A number strip from 95 to 120 for the board (lesson 1), and a 0 to 120 counting track or the class number line if you have one.
- Whiteboards and pens, or paper and pencils, for drawings and number sentences.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week:

- "Count on from 97. I'll tell you when to stop." Stop at 112. (98, 99, 100, 101 ... 112; saying 97 first is fine too.) Then: "Write one hundred and four." (104, not 1004.)
- Show 4 towers of ten and 7 loose cubes. "How many cubes?" (47.) Then: "Show me 63 with cubes." (6 towers of ten and 3 loose cubes.)
- "How many tens and ones are in 16? In 50?" (1 ten and 6 ones; 5 tens and 0 ones.)
- Show the cards 58 and 85. "Which is greater? Put a symbol between them." (85 is greater; 58 < 85, because 5 tens is less than 8 tens.)
- "What is 10 more than 46? What is 10 less than 46?" (56; 36.)
- "What is 70 - 40?" (30: 7 tens take away 4 tens is 3 tens.)
- "What is 34 + 8? Show me with cubes or a drawing." (42: 4 + 8 = 12, a new ten and 2 ones, so 3 tens + 1 ten = 4 tens and 2 ones.) "What is 25 + 30?" (55.)

Pointing to cubes, digit cards or the chart, sign, writing, or a communication device all count as answers. Record yes, with help or not yet for each part.

Sources

- Common Core State Standards, Grade 1 Number and Operations in Base Ten (read to check what is usually expected in grade 1, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/NBT/
- 1.NBT.A.1 count to 120 starting at any number less than 120, and read and write numerals in that range; 1.NBT.B.2 the two digits of a two-digit number are amounts of tens and ones, with the special cases (a) 10 as a bundle of ten ones called a "ten", (b) 11 to 19 as a ten and one to nine ones, (c) 10, 20 ... 90 as one to nine tens and 0 ones; 1.NBT.B.3 compare two two-digit numbers using the meaning of the digits, recording with >, = and <; 1.NBT.C.4 add within 100, including a two-digit number and a one-digit number, and a two-digit number and a multiple of 10, with models, drawings and place-value strategies, adding tens and tens, ones and ones, and sometimes composing a ten; 1.NBT.C.5 find 10 more or 10 less mentally, without counting, and explain the reasoning; 1.NBT.C.6 subtract multiples of 10 in the range 10 to 90 from multiples of 10, with a positive or zero answer.
- Common Core, Grade 1 introduction (four critical areas, the second being whole number relationships and place value): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/introduction/

Known gaps and cautions

- Taking a one-digit number or a two-digit number away from a two-digit number (for example 42 - 7 or 56 - 23) is not taught here; only taking away multiples of ten. Those come in grade 2.
- Adding two two-digit numbers that are not multiples of ten (34 + 25) is not core; it appears only as a stretch task in lesson 5.
- Hundred charts differ: some start at 0, and some count upwards from the bottom. This unit uses 1 to 100 with 1 at the top left. If your class uses another chart, change "down a row" and "up a row" to match.
- Calling > and < a crocodile's mouth that "eats the bigger number" is a tradition some classes use, not part of the source. This unit asks children to read the symbols as words ("is greater than", "is less than"); the crocodile picture is optional.
- Fluency with 10 more and 10 less is expected; fluency with adding within 100 is not.
- Peer check: Gemini 4 Argon did a bounded text, link, timing and answer-key check at 944d536 (not an educator or accessibility review). Links, counts, timings and keys were correct. Its notes (some side-by-side builds in lessons 1 and 3 needed more than 100 cubes; the one-to-one check did not cover writing numbers past 100 or outcome 3; digit cards had no tactile marking) were fixed in the follow-up commit 096f355 (see PEERCHECKS.md), which Argon also diff-checked the same day.
- The lesson order and timings are my choice, not taken from a source.

Lesson 1: Grouping in tens and counting to 120

Unit overview - Next: Lesson 2

About 40 minutes.

Goal

Children count a large collection by grouping it into tens, say how many tens and ones there are, and count on past 100 up to 120.

Materials

- 100 linking cubes per pair.
- A collection of about 30 to 60 small objects per pair (cubes, large buttons or counters, too large to swallow), each on a tray or sheet of paper.
- Optional: about 60 craft sticks per pair, with paper strips and tape for wrapping bundles of ten.
- A number strip from 95 to 120 on the board, and a counting track or class number line if you have one.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count together in ones from different starting numbers: "Start at 28 and count to 35." "Start at 56 and count to 64." Stop at the decade changes (29 to 30, 59 to 60) and ask: "What comes after 39?" (40.) Children who prefer not to say the numbers aloud can point along the number line or tap a finger for each number.

✎ Main activity (30 min)

1. **How many? (10 min).** Give each pair a collection of 30 to 60 objects. "Guess how many, then count them. How can you count so you don't lose track?" Let pairs try their own way first. Then show one way: make groups of ten (in lines, or as towers of linking cubes) and count the groups "10, 20, 30, 40", then the loose ones "41, 42, 43". Pairs count their collection again in tens and write the number. For example 4 groups of ten and 3 more is 43.
2. **Make a ten (10 min).** "Ten ones can become one ten." Each pair snaps cubes into towers of ten. If you have craft sticks, pairs wrap groups of ten sticks with a paper strip to make bundles. The teacher shows some towers and loose cubes and pairs say how many:
    - 2 towers and 6 cubes (26)
    - 5 towers and 0 cubes (50)
    - 1 tower and 8 cubes (18)

    - 7 towers and 4 cubes (74) Then pairs build numbers the teacher says: 35 (3 towers, 5 cubes); 60 (6 towers, no cubes); 19 (1 tower, 9 cubes).
3. **Past 100 (10 min).** Count from 95 to 120 together, pointing to the strip on the board: "...98, 99, 100, 101..." Show that the numbers after 100 go on like the numbers after 0: 101, 102, 103 ... 110, 111 ... 120. Pairs take turns: one says a start number between 95 and 110, the other counts on five numbers (for example 107: 108, 109, 110, 111, 112). Children write two of the numbers they said on a whiteboard (such as 104 and 111).

✓ Discussion questions (with answers)

- Why is counting in tens quicker than counting in ones? We count 4 groups instead of 40 single objects, and if we lose our place we only recount one group.
- Is a tower of ten cubes one thing or ten things? Both: it is ten ones, and we can also count it as one ten.
- What comes after 109? 110. (A common answer is "200"; see teacher notes.)

✓ Closing check (5 min, with answers)

Show 3 towers of ten and 6 loose cubes. "How many?" (36.) "Count on from 98 to 103." (99, 100, 101, 102, 103.)

Vocabulary

ten, tens, ones, group, bundle, tower, count on, hundred

Differentiation

- **Support:** collections up to 30; count to 50 instead of 120.
- **Stretch:** "I have 9 towers and 12 loose cubes. How many cubes?" (102: the 12 loose cubes make another tower and 2, so 10 towers and 2.) This needs more than one pair's 100 cubes: join with another pair, or draw lines for tens and dots for ones.
- Children with low vision or who are blind can count by touch: towers of ten feel different from loose cubes. Children can show answers by building with cubes, pointing to the number line, writing, signing, or using a communication device.

⚠ Safety

Use objects too large to swallow. Keep craft sticks on the tables, and no rubber bands (latex allergies, and they flick).

Teacher notes and misconceptions

- Some children count the towers in ones ("1, 2, 3, 4") and then the loose cubes ("5, 6, 7"), giving 7 instead of 43. Ask them to count one tower cube by cube, then say "so this tower is 10".

- Past 100, children often say "200" after 109, or write 101 as "1001" (one hundred, then one). Say and write the numbers together, and point out that 101 has three digits, like 100.

Lesson 2: Tens and ones on a place-value mat

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 40 minutes.

Goal

Children show two-digit numbers as tens and ones on a mat, explain that 11 to 19 are a ten and some ones and that 10, 20 ... 90 are tens with no ones, and say what each digit means.

Materials

- 100 linking cubes per pair, with some already snapped into towers of ten.
- A place-value mat per pair (tens on the left, ones on the right; card ridge down the middle and a rough sticker on the tens side).
- Two sets of digit cards 0 to 9 per pair, large print with a raised outline.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Show 2 towers and 4 loose cubes. "How many?" (24.) Put the towers on the tens side of a large mat and the loose cubes on the ones side, then put the digit card 2 under the tens side and 4 under the ones side. "The 2 tells us 2 tens. The 4 tells us 4 ones."

✎ Main activity (30 min)

1. **Teen numbers (10 min).** "Show 13 on your mat." (1 tower, 3 cubes.) Pairs show 11 to 19 in turn, putting digit cards under each side, and say the sentence "13 is 1 ten and 3 ones." Ask: "What is the same about all these numbers?" (They all have 1 ten; the first digit is always 1.)
2. **Decades (10 min).** "Show 30." (3 towers, no loose cubes.) "What digit goes on the ones side?" (0, because there are no ones.) Pairs show 20, 50, 80 and 90, and say "50 is 5 tens and 0 ones." Then ask: "How many tens is 100?" (10 tens.) Pairs check by putting 10 towers together.
3. **Same digits, different numbers (10 min).** Pairs show 27 and then 72 (2 tens and 7 ones; 7 tens and 2 ones). "Are they the same?" (No: 72 has many more cubes.) Then one child picks two digit cards and puts them on the mat; the partner builds the number with cubes and says it. Swap roles. Examples to check: 4 and 6 (46 or 64), 9 and 1 (91 or 19), 5 and 0 (50, or 05, which is just 5).

✓ Discussion questions (with answers)

- In 46, what does the 4 mean? 4 tens, which is 40.

- Why is there a 0 in 60? To show there are no ones; without it the number would be 6.
- Is 19 more like 10 or like 90? Like 10: it has only 1 ten (19 is 1 ten and 9 ones).

✓ Closing check (5 min, with answers)

"Show 52 on your mat." (5 towers, 2 cubes.) "What does the 5 mean?" (5 tens, 50.) "How many tens and ones in 80?" (8 tens, 0 ones.)

Vocabulary

place value, tens, ones, digit, tens side, ones side, teen numbers

Differentiation

- **Support:** numbers to 30; one digit card set and a ready-made tower per ten.
- **Stretch:** "Show 34 a different way, using only 2 towers." (2 towers and 14 loose cubes.)
- The ridge and the rough sticker let children tell the two sides of the mat apart by touch, so colour is never the only clue. Children can answer by pointing, building, signing, or using a communication device.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- The teen number names are irregular: "thirteen" says the 3 first, but the 1 ten is written first. Children often write 13 as 31. Saying "1 ten and 3 ones" before writing helps.
- Some children read the digits but not their value ("46 is a 4 and a 6"). Keep asking "4 what?" until the answer is "4 tens".

Lesson 3: Comparing two-digit numbers

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 40 minutes.

Goal

Children compare two two-digit numbers by looking at the tens first and then the ones, and record the result with the symbols >, = and <.

Materials

- 100 linking cubes per pair, with towers of ten.
- A place-value mat per pair.
- Two sets of digit cards 0 to 9 per pair, large print with a raised outline.
- Symbol cards >, < and = per pair, with raised outlines.
- Optional: a toy or home-made balance.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Build 34 and 43 side by side with towers and cubes. "Which has more cubes? How can you tell quickly?" (43: it has 4 tens, and 34 has only 3 tens. The ones do not matter here, because one ten is more than any number of ones up to 9.)

✎ Main activity (30 min)

1. **Tens first, then ones (10 min).** Pairs build each pair of numbers and say which is greater:
    - 36 and 32 (36; same tens, so look at the ones: 6 is more than 2)
    - 50 and 49 (50; 5 tens is more than 4 tens, even though 49 has 9 ones)
    - 41 and 14 (41; 4 tens is more than 1 ten)
    - 28 and 28 (they are equal) Each time, children say the sentence: "36 is greater than 32."
2. **The symbols (10 min).** Write 36 > 32 and read it: "36 is greater than 32." Write 32 < 36 and read it: "32 is less than 36." The pointed end points to the smaller number; the open side faces the greater number. (Some classes call the symbol a crocodile's mouth that opens towards the bigger number; that picture is optional.) Pairs put a symbol card between number cards and read the sentence aloud or sign it:
    - 63 and 36 (63 > 36)
    - 47 and 49 (47 < 49)
    - 80 and 80 (80 = 80)
    - 15 and 51 (15 < 51)

3. **Make the bigger number (10 min).** Each child in a pair takes two digit cards without looking and arranges them to make the greatest number they can (with 3 and 7, make 73). Partners put their numbers side by side, place the right symbol between them and read the sentence. The child with the greater number keeps a cube; play several rounds. Tell children that "0" cannot go first in this game, since 05 is just 5.

✓ Discussion questions (with answers)

- Why do we look at the tens first? Because one ten is more than all the ones in a number (at most 9), so the number with more tens is greater.
- When do we need to look at the ones? When the tens are the same, as in 36 and 32.
- You have the cards 3 and 7. What is the greatest number you can make, and the smallest? 73 and 37.

✓ Closing check (5 min, with answers)

"Which symbol goes between 63 and 36?" (>, 63 is greater.) "Between 47 and 49?" (<, 47 is less.) "Read this: 28 = 28." ("28 is equal to 28.")

Vocabulary

greater than, less than, equal to, compare, symbol, tens first

Differentiation

- **Support:** compare numbers within 30, and build both numbers every time before choosing a symbol. Use a balance to show "equal" if you have one.
- **Stretch:** "Find all the numbers that make this true: 4□ > 46." (47, 48, 49.)
- Raised symbol cards let children feel which way the symbol points. Children can answer by placing a card, pointing, signing, or using a communication device.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Every number built in this lesson fits in one pair's 100 cubes (the biggest side-by-side build, 50 and 49, uses 99).
- Children who look at the ones first may say 49 is greater than 50 ("9 is bigger than 0"). Build both: 4 towers and 9 cubes next to 5 towers.
- The symbols are often reversed. Reading the whole sentence aloud, from left to right, is a better check than remembering which way the symbol faces.

Lesson 4: Ten more, ten less, and adding and taking tens

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 40 minutes.

Goal

Children find 10 more and 10 less than a two-digit number without counting in ones, and add and take away multiples of ten.

Materials

- A 1 to 100 chart per pair (ten rows of ten, 1 at the top left) and a large one for the board, with three counters per child.
- 100 linking cubes per pair, with towers of ten.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count in tens together from 0 to 100, then from 3: "3, 13, 23, 33 ..." up to 93. "What changes and what stays the same?" (The tens digit goes up by one; the ones digit stays 3.)

✎ Main activity (30 min)

1. **Patterns on the chart (10 min).** Pairs put a counter on 24 and then on the number just below it (34) and just above it (14). "What happens when we move down one row?" (The number goes up by 10.) "Up one row?" (It goes down by 10.) Pairs try from 57 (down 67, up 47) and from 81 (down 91, up 71).
2. **10 more, 10 less (10 min).** Build 34 with towers and cubes. "Add one tower. How many now?" (44.) "Take away one tower from 34." (24.) Only the tens change. Pairs answer without counting in ones, then check on the chart or with cubes:
    - 10 more than 46 (56)
    - 10 less than 71 (61)
    - 10 more than 89 (99)
    - 10 less than 15 (5)
    - 10 more than 90 (100)
3. **Adding and taking tens (10 min).** "30 + 40. 3 tens and 4 tens make 7 tens, so 70." Pairs use towers only:
    - 50 + 20 (70)
    - 30 + 60 (90)
    - 80 - 30 (50)
    - 90 - 40 (50)

    - 60 - 60 (0) Each time ask children to say it in tens: "8 tens take away 3 tens is 5 tens."

✓ Discussion questions (with answers)

- Why does only the tens digit change when we add 10? We add one ten and no ones, so the ones stay the same.
- What is 10 less than 15, and where is it on the chart? 5. It is in the same column as 15, one row up.
- How is 7 - 4 like 70 - 40? 7 ones take away 4 ones is 3 ones; 7 tens take away 4 tens is 3 tens, or 30.

✓ Closing check (5 min, with answers)

"What is 10 more than 52?" (62.) "10 less than 52?" (42.) "What is 70 - 20?" (50.)

Vocabulary

10 more, 10 less, row, column, hundred chart, multiple of ten

Differentiation

- **Support:** use towers and cubes for every question before the chart; numbers to 50.
- **Stretch:** "What is 20 more than 37? 30 less than 85?" (57; 55.)
- Large-print charts with a raised dot on each number in the 10s column help children with low vision or who are blind find the rows. Counters mark numbers, so colour is never needed. Children can answer by pointing, building, signing, or using a communication device.

⚠ Safety

Use cubes and counters too large to swallow.

Teacher notes and misconceptions

- Some children count on ten ones every time ("46, 47, 48 ... 56"). That gives the right answer but misses the point; ask "What changed?" and build it with one tower.
- On the chart, the numbers that end in 0 sit at the right-hand end of each row, so 30 is in the row with 21 to 29, not with 31. Children may expect 30 to start a new row.

Lesson 5: Adding within 100

Unit overview - Previous: Lesson 4

About 40 minutes.

Goal

Children add a multiple of ten or a one-digit number to a two-digit number within 100, putting tens with tens and ones with ones, and make a new ten when the ones add up to 10 or more.

Materials

- 100 linking cubes per pair, with towers of ten.
- A place-value mat per pair.
- Two sets of digit cards 0 to 9 per pair, large print with a raised outline.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Quick questions from lesson 4: "10 more than 63?" (73.) "40 + 30?" (70.) "What is 7 + 5?" (12.) "How many tens and ones is 12?" (1 ten and 2 ones.)

✎ Main activity (30 min)

1. **Adding tens (10 min).** "23 + 30. Build 23 on your mat. Add 3 towers to the tens side. How many tens now? How many ones?" (5 tens, 3 ones: 53.) Only the tens change. Pairs build and write the number sentence:
    - 46 + 20 (66)
    - 58 + 40 (98)
    - 15 + 70 (85)
2. **Adding ones (10 min).** "32 + 5. Build 32, add 5 loose cubes to the ones side." (3 tens, 7 ones: 37.) Only the ones change. Pairs try:
    - 61 + 7 (68)
    - 44 + 4 (48)
    - 83 + 6 (89)
3. **Making a new ten (10 min).** "27 + 5. Build 27, add 5 loose cubes. How many ones now?" (12.) "Can we make a new tower?" Snap 10 of the ones into a tower and move it to the tens side: now 3 tens and 2 ones, 32. Write it as 7 + 5 = 12, so 20 + 12 = 32. Pairs try, making a new tower each time:
    - 38 + 6 (44)
    - 45 + 9 (54)
    - 56 + 4 (60; the new ten leaves no ones)

✓ Discussion questions (with answers)

- In 23 + 30, why do we add the 3 tens to the tens side, not the ones side? 30 is 3 tens; 23 + 3 would be 26, which is a different sum.
- When do we need to make a new ten? When the ones add up to 10 or more, as in 27 + 5, where 7 + 5 = 12.
- Why is 56 + 4 exactly 60? 6 + 4 = 10 ones, which is a new ten, so we have 6 tens and no ones.

✓ Closing check (5 min, with answers)

"36 + 7. Show me with cubes or a drawing." (43: 6 + 7 = 13, a new ten and 3 ones, so 4 tens and 3 ones.) "52 + 30?" (82.)

Vocabulary

add, tens, ones, make a new ten, number sentence, sum

Differentiation

- **Support:** questions without a new ten first; totals up to 50; build every question with cubes.
- **Stretch:** "34 + 25. Add the tens, then the ones." (30 + 20 = 50, 4 + 5 = 9, so 59.) Then: "27 + 35." (50 + 12 = 62.)
- Children can draw tens as lines and ones as dots instead of using cubes. Children who are blind or have low vision can build on the tactile mat. Answers can be given by pointing to digit cards, building, signing, or a communication device.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- A common error in 23 + 30 is adding the 3 to the ones (answer 26) or writing "2330". Saying "30 is 3 tens" before building helps.
- In 27 + 5, some children write 212 ("2 tens and 12 ones"). Making the new tower with real cubes, and moving it to the tens side, shows why the answer is 32.
- Use the last part of the week for the one-to-one check in the unit overview.

Grade 1 Mathematics: Measuring Length and Putting Shapes Together (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 9 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

Back to the grade 1 year plan - Term 3. Builds on term 2: Tens and ones to 100

Big idea

To compare lengths fairly, we line the ends up. When two things cannot be put side by side, a third thing (a piece of string) can carry one length to the other. To say how long something is, we lay copies of one small unit end to end, with no gaps and no overlaps, and count them. Shapes have a few things that make them that shape (a triangle is closed and has three straight sides), and things that do not matter (colour, size, which way it is turned). Shapes can be put together to make new shapes.

Outcomes

By the end of the unit most children can:

1. Put three objects in order from shortest to longest, lining up the ends.
2. Compare two lengths that cannot be put side by side by using a third object, and explain the result in words ("the string was the same as the table, and the rug was longer than the string, so the rug is longer").
3. Measure a length by laying equal units end to end with no gaps or overlaps, and say the length as a number of units ("8 cubes long").
4. Say whether a shape is a triangle, square or rectangle using its sides and corners, not its colour, size or which way it is turned, and build one with craft sticks or draw one.
5. Put two or more shapes together to make a new shape (two triangles make a square), and use that new shape to make a bigger one.

Prior knowledge

The kindergarten units on comparing (longer, shorter, taller) and naming flat shapes, and counting to 20 and beyond from terms 1 and 2.

Lessons

Each lesson is about 40 minutes: a short whole-class start, paired work and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so measuring and shape sorting should come back in short daily practice (5 minutes).

1. Shortest to longest

2. Comparing with a piece of string
3. Measuring with equal units
4. What makes a triangle a triangle?
5. Putting shapes together

Materials (whole class unless noted)

- Lesson 1: two pencils of different lengths; per pair, two sets of three stiff card strips (set 1: 8, 14 and 20 cm; set 2: 11, 17 and 24 cm), each strip marked with a letter (A, B, C) and the same number of raised dots (1, 2, 3); a hardback book or ruler as an edge to line ends up against; per child, a recording sheet and glue stick; 6 spare strips and a large book to stand up as a screen for the closing check (see lesson 1).
- Lesson 2: masking tape; six measuring stations prepared before the lesson (see lesson 2); per pair, one piece of string about 1.5 m long, cut by an adult, and two clothes pegs; per child, a recording sheet and two small answer cards (a table picture and a rug picture); two picture cards for the closing check (see lesson 2); the teacher's answer key for the stations.
- Lesson 3: per pair, 30 linking cubes all the same size (the cubes from term 2), 10 unit cards (identical card rectangles, each exactly as long as two cubes), and three card strips to measure, cut to exactly 6, 8 and 10 cubes long; per child, a recording sheet; a tray with four demonstration strips (one with a gap, one with an overlap, one with mixed cubes and unit cards, and one measured correctly).
- Lesson 4: per pair, a set of 24 shape cards cut from card (so edges can be felt): 8 triangles of different sizes, colours and turns, 4 squares, 4 rectangles, and 8 "not quite" shapes (a three-sided shape with a gap, a three-sided shape with one curved side, a four-sided shape that is not a rectangle, a circle, a half-circle, and three other shapes with more than four sides); four large shape cards for the closing check, hand-sized copies of them and a small blue triangle (see lesson 4); a sorting mat with two areas labelled "triangle" and "not a triangle" (words, a drawn triangle and a raised outline); per child, 8 craft sticks (checked for splinters) and a whiteboard and pen.
- Lesson 5: per pair, a bag of card shapes: 8 right-angled triangles (two equal 5 cm sides), 4 squares (5 cm sides), 2 rectangles (5 cm by 10 cm), 4 half-circles (10 cm across) and 4 quarter-circles (5 cm radius); 6 outline cards to fill (see lesson 5); per child, a blank sheet, pencil, whiteboard and pen; a larger set of the card shapes for the teacher to show.
- Assessment: one set of lesson 1 strips, the lesson 2 picture cards, a lesson 3 strip and cubes, 6 shape cards, 8 craft sticks and 4 triangles from lesson 5.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 6 minutes per child, so about 2.5 hours for 25 children, spread over the fourth week):

- Give three strips from set 2, ends not lined up. "Put them in order from shortest to longest." (11, 17, 24 cm. Watch whether the child lines up the ends.)
- Show the two lesson 2 picture cards and say: "Sam's string was just as long as the table. The string was shorter than the rug. Which is longer, the table or the rug?" (The rug.)
- Give the 8-cube strip and loose cubes. "How long is this strip in cubes?" (8 cubes, laid end to end with no gaps or overlaps.) Then show the gap strip from lesson 3: "Is this measured right? Why not?" (No: there is a gap, so the count is too small.)
- Give 6 shape cards (2 triangles turned different ways, a square, a rectangle, the three-sided shape with a gap, and the shape with a curved side). "Give me all the triangles." (The 2

triangles only.) "Why is this one not a triangle?" (It is not closed / one side is not straight.) "Which card is a rectangle but not a square?" (The rectangle.)
- Give 4 craft sticks. "Make a triangle." Then, with 4 sticks, "Make a square." (A closed shape with three straight sides; then four equal sides and four square corners. Drawing the shapes on a whiteboard counts too.)
- Give 4 right-angled triangles. "Make a square." (Long sides together.) "Now make a rectangle using that square." (Make a second square from the other two triangles and put the two squares side by side.)

Pointing, moving the pieces, sign, writing, or a communication device all count as answers. Record yes, with help or not yet for each part.

Sources

- Common Core State Standards, Grade 1 Measurement and Data (read to check what is usually expected in grade 1, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/MD/
- 1.MD.A.1 order three objects by length; compare the lengths of two objects indirectly by using a third object. 1.MD.A.2 express the length of an object as a whole number of length units by laying multiple copies of a shorter object end to end; the length is the number of same-size units that span it with no gaps or overlaps; limited to objects spanned by a whole number of units.
- Common Core, Grade 1 Geometry: https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/G/
- 1.G.A.1 distinguish defining attributes (for example, triangles are closed and three-sided) from non-defining attributes (colour, orientation, overall size); build and draw shapes with defining attributes. 1.G.A.2 compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles and quarter-circles) to make a composite shape, and compose new shapes from the composite shape. A footnote says children should use transitivity to make indirect comparisons but need not use the term.
- Common Core, Grade 1 introduction (four critical areas, the third being linear measurement as iterating length units and the fourth composing and decomposing shapes): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/introduction/

Known gaps and cautions

- Peer check: Gemini 4 Argon did a bounded text, link, timing, number and shape-geometry check at 1ba989a (not an educator or accessibility review). Its notes (the assessment did not check building or drawing shapes, or making a bigger shape from a made shape; station lines in lesson 2 needed feelable marks at both ends, not only the start) were fixed in 049cf5e, which Gemini 4 Argon also diff-checked the same day (see PEERCHECKS.md). This is a peer check by an AI agent, not an educator review.
- Not covered here: telling time (1.MD.B.3), data with up to three categories (1.MD.C.4; the term 3 computing unit in the year plan sorts and counts with a picture chart) and halves and quarters (1.G.A.3). A school following the Common Core would need to teach these elsewhere.
- Three-dimensional shapes (cubes, cylinders, cones) are named in 1.G.A.2 but this unit uses flat shapes only.
- Standard units (centimetres, inches) are not taught; the strip lengths in centimetres are for the teacher to cut them.

- The unit never measures or compares children's bodies (heights, hands, feet). That is a choice made here, not from the source, so that nobody's size is compared in front of the class.
- The strip lengths, station objects and shape sets are examples, not from a source.

Lesson 1: Shortest to longest

Unit overview - Next: Lesson 2

About 40 minutes.

Goal

Children put three objects in order by length and explain that the ends must be lined up to compare fairly.

Materials

- Two pencils of different lengths (for example 10 cm and 15 cm).
- Per pair: two sets of three stiff card strips (set 1: 8, 14 and 20 cm; set 2: 11, 17 and 24 cm). Mark each strip with a letter (A, B, C, in a mixed order so the letters do not give the answer) and the same number of raised dots (one dot for A, two for B, three for C).
- Per pair: a hardback book or ruler to line the ends up against.
- Per child: a recording sheet with three empty boxes labelled "shortest", "in between" and "longest", and a glue stick.
- For the closing check: 6 spare strips (two sets of three, made like set 1 and set 2) and a large book to stand up as a screen.

Steps

✎ Opening (5 min)

Hold the two pencils upright in one fist so the short pencil sticks up higher than the long one (its bottom end is hidden lower down in your hand). "Which pencil is longer?" Many children will say the one that sticks up. Open your hand and lay both pencils on the table with their ends against the edge of a book. "Now which is longer?" Agree: **to compare lengths, line up the ends.**

✎ Main activity (30 min)

1. Model with set 1: put the three strips against the book edge, ends touching it, and run a finger along to the other ends. "Which end sticks out furthest? That strip is the longest." Say the order aloud: shortest, in between, longest.
2. Pairs order set 1 against their own book edge. One child lines up the ends; the other says the order. Swap roles and do set 2.
3. Each child glues one set on their recording sheet, shortest to longest.
4. Stretch: "Mix both sets together. Can you put all six strips in order?" (8, 11, 14, 17, 20, 24 cm.)

✓ Discussion questions (with answers)

- "Why did the short pencil look longer at the start?" (Its ends were not lined up with the other pencil's ends.)
- "B is longer than A. C is longer than B. Is C longer than A?" (Yes.)
- "If I turn the longest strip round, is it still the longest?" (Yes; turning it does not change its length.)

✓ Closing check (5 min, with answers)

Behind the standing book, lay out three of the spare strips with their ends not lined up, then lift the book away. Each child shows on their fingers (1, 2 or 3 dots) which strip is the longest, all at the same time, then which is the shortest. Repeat once with the other spare set. (Children who line the ends up in their heads or ask to move the strips are doing the right thing; let them move them.)

Vocabulary

long, longer, longest; short, shorter, shortest; in between; length; line up the ends

Differentiation

- Support: use two strips before three, with a big difference in length.
- Stretch: the six-strip order above; or order three classroom objects (a crayon, a glue stick, a pencil case) chosen by the child.
- Blind or low-vision children: line the strips up against the book edge by touch and feel which end sticks out furthest; the raised dots name the strips.
- Children who cannot use their fingers easily can nudge strips with a palm or say/point to the order; a partner can line up the ends while they direct.
- For the closing check, children who do not use fingers can point, nod at the right strip as the teacher touches each one, or sign; blind children feel the three strips after the book is lifted.
- Colour is never the only clue; letters and dots name the strips.

⚠ Safety

Card strips only; no scissors for children in this lesson.

Teacher notes and misconceptions

- Children often compare only one end (whichever sticks out at the top or right). The pencil opening is there to make that visible.
- "Longest" and "tallest" are the same idea; "tallest" is used when the object stands up.

Lesson 2: Comparing with a piece of string

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 40 minutes.

Goal

Children compare two lengths that cannot be put side by side by carrying one length to the other with a piece of string, and explain the result.

Materials

- Masking tape and six measuring stations, set up before the lesson. Each station has two lengths in different places, marked A and B with a letter and a raised tape tab at both ends of each line (a folded tab at the start and a small block or a dot of tape at the finish), so the ends can be felt. An example set (not from a source; change it to fit your room):
    1. Two floor lines in different corners: A 100 cm, B 120 cm.
    2. A line on a table top (A, 60 cm) and a line on the floor (B, 50 cm).
    3. A line on a wall at about 80 cm from the floor, so it can be reached from a wheelchair (A, 70 cm), and a floor line (B, 70 cm). These are the same length.
    4. Two floor lines, one straight across (A, 80 cm) and one slanting (B, 90 cm).
    5. The width of the teacher's table (A) and the width of a cupboard (B). Choose two things that are each shorter than the string.
    6. The width of the classroom door (A) and the width of a bookshelf (B).
- The teacher's answer key: write the answers for stations 5 and 6 after checking them with string before the lesson.
- Per pair: one piece of string about 1.5 m long, cut by an adult, and two clothes pegs.
- Per child: a recording sheet with a row per station: "A is longer", "B is longer", "the same" (words and simple pictures), to circle.
- Two picture cards for the closing check: a table with a string lying along it, just as long, and a rug with the same string lying on it, shorter than the rug.
- Per child: two small answer cards, one with a table picture and one with a rug picture.

Steps

✎ Opening (5 min)

"I want to know if this table is longer than the rug. Can I pick up the table and put it next to the rug?" (No, it is too heavy.) "In lesson 1 we lined the ends up. What could we do now?" Take ideas. Show the string: lay it along line A at station 1 with one end at the start tab, and clip a peg on the string exactly where line A ends. "Now the string remembers how long line A is." Carry it to line B.

✎ Main activity (30 min)

1. At station 1, lay the string along line B from its start tab. "Does the peg come before the end of B, at the end, or past it?" (Before the end: B is longer.) Say the reasoning aloud together: "The string is as long as A. B is longer than the string. So B is longer than A."
2. Pairs visit three or four stations. One child holds the string's end at the start tab; the other lays it along and clips the peg. They swap roles at the next station. Each child circles the answer on their sheet.
3. As you go round, ask each pair to say the reasoning sentence for one station: "The string is as long as ... . is longer / shorter / the same as the string. So ... ."
4. Stretch: "Use the string to find something in the room that is about as long as line A at station 1." (Answers vary; check by laying the string along it.)

✓ Discussion questions (with answers)

- Station 1: B is longer. Station 2: A is longer. Station 3: the same. Station 4: B is longer. Stations 5 and 6: see your key.
- "At station 3, one line is on the wall and one is on the floor. Does that change which is longer?" (No; where a line is, or which way it points, does not change its length.)
- "What would go wrong if the string's end did not start at the start tab?" (The peg would not show the real length; it is like not lining up the ends.)

✓ Closing check (5 min, with answers)

Show the two picture cards and say: "Sam's string was just as long as the table. Then Sam put the string on the rug. The string was shorter than the rug. Which is longer, the table or the rug?" Children hold up one answer card at the same time. (The rug.) Then: "If the string had been longer than the rug, which would be longer?" (The table.)

Vocabulary

compare; just as long as; the same length; longer than; shorter than; start; end

Differentiation

- Support: begin with station 1 or 4, where the difference is 10 to 20 cm; an adult can hold the start end.
- Stretch: put three lengths in order with the string: line A at station 4 and both lines at station 1. (Station 4 line A is shortest, then station 1 line A, then station 1 line B.)
- Blind or low-vision children: feel the folded start tab and the block or tape dot at the finish; feel where the peg sits against the finish.
- Wheelchair users: station 3's wall line and table lines are reachable; for floor lines, a partner lays the string while the child directs and checks the peg.
- Children who use sign or a device give the reasoning sentence that way; a partner can say it while they point to each part.

⚠ Safety

- Adults cut and hand out the string; no scissors for children.
- String goes along lines and objects only, never round fingers, wrists or necks. Collect it at the end of the lesson.
- Floor stations should be away from doors and walkways so nobody trips.

Teacher notes and misconceptions

- The lengths of stations 1 to 4 are chosen so the answers are clear (at least 10 cm apart) except station 3, where the point is that two lengths in different places can be the same.
- Some children think a slanting line is longer because it "goes further". Station 4 lets them check.
- The standards expect children to use this kind of reasoning ("if A is the same as the string and B is longer than the string, B is longer than A") without learning a name for it.

Lesson 3: Measuring with equal units

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 40 minutes.

Goal

Children measure a length by laying equal units end to end, with no gaps and no overlaps, and say the length as a number of units.

Materials

- Per pair: 30 linking cubes all the same size (the cubes from term 2).
- Per pair: 10 unit cards: identical card rectangles, each exactly as long as two cubes joined together.
- Per pair: three card strips to measure, cut to exactly 6, 8 and 10 cubes long and labelled with a letter and raised dots (P, one dot; Q, two dots; R, three dots).
- A tray with three demonstration strips measured wrongly: one with a gap between two cubes, one with two cubes overlapping (one cube pushed partly on top of another), and one with cubes and unit cards mixed; and a fourth strip measured correctly with cubes, for the closing check.
- Per child: a recording sheet with a row for each strip: "Strip P is ... cubes long" and "Strip P is ... unit cards long".

Steps

✎ Opening (5 min)

Lay a strip on the table and put loose cubes along it with gaps between them. "I counted 4 cubes, so it is 4 cubes long. Do you agree?" Take ideas. Push the cubes together with no gaps, and keep adding cubes until they reach the end. Count again. Agree the rules: **start at one end, no gaps, no overlaps, all the units the same size, stop at the other end.**

✎ Main activity (30 min)

1. Pairs measure strips P, Q and R with cubes. One child lays the cubes from the start end; the other checks for gaps and overlaps and counts. Swap for each strip. Children write or say the number (P 6, Q 8, R 10).
2. Pairs measure the same strips with unit cards (P 3, Q 4, R 5).
3. Show the tray. For each demonstration strip, ask: "What went wrong? Will the count be too big or too small?" (Gap: some of the strip is not covered, so the count is too small. Overlap: two units share the same bit of strip, so the count is too big. Mixed units: the number does not tell us how long, because the units are not the same.)

4. Stretch: "Strip R is 10 cubes long but only 5 unit cards long. Why are the numbers different?" (A unit card is as long as two cubes, so you need fewer of them.) Or: lay P, Q and R end to end and measure all three together (24 cubes).

✓ Discussion questions (with answers)

- "Strip Q is 8 cubes long. If Ali counts 7, what might have happened?" (A gap, or Ali did not start at the end.)
- "If Ben counts 9, what might have happened?" (An overlap, or a cube past the end.)
- "Could we measure with a cube and a pencil and a crayon?" (No; the units must all be the same size, or the number does not mean anything.)

✓ Closing check (5 min, with answers)

Hold up each demonstration strip from the tray, one at a time, and a fourth strip measured correctly. Children show thumbs up (measured right) or thumbs down (measured wrong) at the same time. (Gap: down. Overlap: down. Mixed: down. Correct: up.) Then ask one child to say what was wrong with the gap strip. (It has a gap, so the count is too small.)

Vocabulary

measure; unit; end to end; gap; overlap; the same size; how long

Differentiation

- Support: measure strip P only, then Q; place a line on the strip where each cube should go (6 boxes drawn on strip P).
- Stretch: the two stretch questions above.
- Blind or low-vision children: feel along the strip for gaps between cubes, and feel that the last cube ends at the strip's end; linking the cubes together removes gaps.
- Children who cannot use their fingers easily can direct a partner ("one more, push it closer") and check by looking or touching with a palm.
- Children who do not use thumbs can hold up a "yes" or "no" card, or nod or shake their head, for the closing check.

⚠ Safety

Collect loose cubes from the floor; nobody should step on them.

Teacher notes and misconceptions

- The strips are cut so they are exactly a whole number of units long; the standard limits grade 1 measuring to this.
- Linking the cubes into a train is a good way to avoid gaps. Some children count the cubes before they have reached the end; ask them to run a finger along to check.

- Comparing cube counts with unit-card counts is a stretch, not an outcome.

Lesson 4: What makes a triangle a triangle?

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 40 minutes.

Goal

Children decide whether a shape is a triangle, square or rectangle by its sides and corners, not by its colour, size or which way it is turned, and build these shapes with craft sticks.

Materials

- Per pair: a set of 24 shape cards cut from card (so the edges can be felt): 8 triangles of different sizes, colours and turns (including long thin ones and upside-down ones); 4 squares; 4 rectangles that are not squares; and 8 "not quite" shapes: a three-sided shape with a gap at one corner, a three-sided shape with one curved side, a four-sided shape that is not a rectangle, a circle, a half-circle, and three shapes with more than four sides.
- Per pair: a sorting mat with two areas labelled "triangle" and "not a triangle" (the words, a drawn triangle and a raised outline of a triangle).
- Four large shape cards for the closing check: an upside-down triangle, a three-sided shape with a gap, a big red triangle, and a three-sided shape with one curved side; a hand-sized copy of each of these four, and a small blue triangle, for the opening and for children who check by touch.
- Per child: 8 craft sticks (checked for splinters), a whiteboard and pen.

Steps

✎ Opening (5 min)

Hold up the big red triangle card, then turn it upside down. "Is it still a triangle?" Then the small blue triangle. "Is this a triangle?" Agree that colour, size and turning do not change what shape it is. Ask: "So what does make a triangle?" Collect ideas and agree on: **closed (no gaps), three straight sides, three corners.**

✎ Main activity (30 min)

1. Pairs sort their shape cards onto the mat: triangle or not a triangle. For each "not a triangle" card with three sides, they say why. (Gap: not closed. Curved side: not all sides straight.)
2. Take the squares and rectangles out of the "not a triangle" pile. Count sides and feel the corners. "Both have four straight sides and four square corners. A square's sides are all the same length." Pairs sort them into squares and rectangles.
3. Building: each child makes a triangle with 3 craft sticks, then a square with 4. "Can you make a rectangle that is not a square?" (Use 6 sticks: two sticks end to end for each long side, one stick for each short side.) Children check each other's shapes against the rules.

4. Children draw one triangle and one rectangle on their whiteboard and show a partner.
5. Stretch: "Make the biggest triangle you can with all 8 sticks." Or: "Draw a triangle that nobody else in the class will draw." (Any closed shape with three straight sides counts.)

✓ Discussion questions (with answers)

- "Mia says this long thin shape is not a triangle because it is too thin. Is she right?" (No: it is closed and has three straight sides, so it is a triangle.)
- "Is a square turned on its corner still a square?" (Yes; turning does not change the shape.)
- "Why is the shape with a gap not a triangle?" (It is not closed.)

✓ Closing check (5 min, with answers)

Hold up the four large closing-check cards, one at a time, and leave them in a row. Each child writes on their whiteboard how many of the four are triangles and everyone holds up together. (2: the upside-down triangle and the big red triangle.) Ask one child why the curved one is not a triangle. (One side is not straight.)

Vocabulary

triangle; square; rectangle; side; corner; straight; curved; closed; turned

Differentiation

- Support: sort only triangles and circles first, then add the "not quite" shapes.
- Stretch: the two stretch tasks above.
- Blind or low-vision children: feel along each edge for straight or curved sides and count corners by touch; the gap shape has a clear gap to feel. For the closing check, give them a hand-sized copy of each card to feel.
- Children who cannot hold craft sticks easily can direct a partner by pointing to where each stick goes.
- Children who cannot write a number can hold up fingers, point to a number card, or sign.
- Colour is never the clue to a shape's name.

⚠ Safety

Check craft sticks for splinters before the lesson; collect them afterwards.

Teacher notes and misconceptions

- Children often think a triangle must have a flat bottom and a point at the top, because that is how pictures usually show it. Many turned and thin triangles in the card set help.
- A square is a special kind of rectangle (it has four square corners). Some classes say this in grade 1 and some leave it for later; it is not assessed in this unit.

Lesson 5: Putting shapes together

Unit overview - Previous: Lesson 4

About 40 minutes.

Goal

Children put two or more shapes together to make a new shape, then use that new shape to make a bigger one.

Materials

- Per pair: a bag of card shapes: 8 right-angled triangles (the two sides next to the square corner are each 5 cm), 4 squares (5 cm sides), 2 rectangles (5 cm by 10 cm), 4 half-circles (10 cm across) and 4 quarter-circles (5 cm radius).
- Per pair: 6 outline cards, each with a shape drawn full size and its outline raised (glue line or wool) so it can be felt:
    1. a square, 5 cm sides;
    2. a rectangle, 5 cm by 10 cm;
    3. a circle, 10 cm across;
    4. a trapezoid, 15 cm along the bottom, 5 cm along the top, 5 cm high;
    5. a big right-angled triangle, 10 cm along each side next to the square corner;
    6. a big square, 10 cm sides.
- Per child: a blank sheet and a pencil, for drawing round a design.
- Per child: a whiteboard and pen (as in lesson 4), for the closing check.
- One set of the card shapes, larger, for the teacher to show (or a visualiser).

Steps

✎ Opening (5 min)

Show two triangles. "What shape could these two triangles make if I put them together?" Take guesses, then put the long sides together: a square. Then: "Two of these squares?" (A rectangle.) "Now we have made a rectangle out of triangles and a square. We made a new shape from a shape we had already made."

✎ Main activity (30 min)

1. Pairs fill outline cards 1, 2 and 3 with shapes from the bag, so that the shapes cover the outline with no gaps and no overlaps. They say which shapes they used.
2. Then cards 4, 5 and 6. Encourage more than one way for each card.
3. Free building: each child makes their own picture or pattern from at least four shapes and draws round it on their sheet. Partners say one shape they can see that was made from two smaller shapes.
4. Opt-in share: children who want to can show their design to the class. Nobody has to.

5. Stretch: "Make a big square from triangles only. How many triangles do you need?" (8 for card 6.) "Make a circle from quarter-circles. How many?" (4.)

✓ Discussion questions (with answers)

Answer key for the outline cards (other correct ways count too; check with the shapes):

- Card 1 square: 2 triangles (or 1 square).
- Card 2 rectangle: 2 squares, 4 triangles, 2 triangles and 1 square, or 1 rectangle.
- Card 3 circle: 2 half-circles, or 4 quarter-circles, or 1 half-circle and 2 quarter-circles.
- Card 4 trapezoid: 1 square in the middle with a triangle on each side, or 4 triangles.
- Card 5 big triangle: 4 triangles, or 1 square and 2 triangles.
- Card 6 big square: 4 squares, 2 rectangles, 8 triangles, or a mix (for example 1 rectangle and 2 squares).
- "Two half-circles make a circle. What do two quarter-circles make?" (A half-circle.)

✓ Closing check (5 min, with answers)

Show outline card 1 and two triangles. "How many triangles fill this square?" Children write the number on a whiteboard, or hold up fingers, at the same time. (2.) Then show outline card 6: "How many small squares fill this big square?" (4.)

Vocabulary

put together; make; new shape; square; rectangle; triangle; trapezoid; half-circle; quarter-circle; circle; no gaps, no overlaps

Differentiation

- Support: start with cards 1 and 3, with the shapes needed set out beside the card.
- Stretch: the two stretch questions above; or make a shape and give a partner its outline to fill.
- Blind or low-vision children: feel the raised outline, then fit shapes inside it by touch; edges of card shapes can be felt meeting.
- Children who cannot place shapes easily can point to where each shape goes while a partner places it, or move shapes by sliding them with a palm.
- The free design is shared only if the child wants to.

⚠ Safety

Card shapes only; no small pieces that need picking up from the floor beyond the usual tidy-up.

Teacher notes and misconceptions

- Some children turn a shape the wrong way and decide "it doesn't fit". Encourage turning and flipping.
- The trapezoid is named here because it appears in the source's list of shapes; children do not need to define it.

Grade 2 Mathematics: Adding and Taking Away Within 100 (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 10 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

Back to the grade 2 year plan - Term 1. Builds on grade 1: Adding and taking away within 20 and Tens and ones to 100

Big idea

Two-digit numbers are tens and ones, so we can add or take away tens with tens and ones with ones. Sometimes ten ones join to make a new ten, and sometimes a ten has to be broken into ten ones. Adding and taking away undo each other, so one can check the other. A drawing with a box for the unknown helps us see what a story problem is asking, even when it has two steps.

Outcomes

By the end of the unit most children can:

1. Add and take away within 20 quickly using a strategy they can name (doubles, near doubles, making ten, or "what do I add?").
2. Add two two-digit numbers by adding tens and tens and ones and ones, using cubes, drawings or the hundred square, including sums where the ones make a new ten (38 + 25 = 63).
3. Take away a two-digit number from a two-digit number, including when a ten has to be broken into ones (52 - 17 = 35), and check the answer by adding.
4. Explain with cubes or a drawing why their method works ("8 ones and 5 ones are 13 ones, which is 1 ten and 3 ones").
5. Solve one-step story problems within 100 of the five kinds (adding to, taking from, putting together, taking apart, comparing), and two-step story problems, using a drawing and a number sentence with a box for the unknown.

Prior knowledge

The grade 1 units: facts within 20 with strategies (fluency within 10 only), tens and ones to 100, 10 more and 10 less, adding a one-digit number or a multiple of ten to a two-digit number. Children still building these can work with numbers to 50, or with sums that do not make a new ten, in every lesson.

Lessons

Each lesson is about 45 minutes: a short whole-class start, paired work with cubes, and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so each method should come back in short daily practice (5 minutes of quick facts or one story problem).

1. Facts within 20: checking and speeding up
2. Adding tens and ones
3. Adding when the ones make a new ten
4. Taking away, breaking a ten, and checking by adding
5. One-step and two-step story problems

Materials (whole class unless noted)

- Linking cubes too large to swallow, per pair: 10 towers of ten and 20 loose cubes (lessons 2 to 5). Straw bundles of ten wrapped with tape (no rubber bands, which may contain latex) work as well.
- Two ten-frames and 20 counters per child (lesson 1), in two colours or two kinds of object so colour is not the only clue.
- A tens-and-ones mat per pair: a sheet with a left column for tens and a right column for ones, with a raised line (glued string) between them (lessons 2 to 5).
- A 1 to 100 hundred square per pair, in large print, with optional raised lines between rows (lessons 2 and 4); one large hundred square for the board.
- Fact cards and four strategy cards (double, near double, make ten, what do I add?) per pair for lesson 1 (see lesson 1).
- Optional: a cup marked "ten" per pair (lesson 3, for children who slide ten loose cubes into it instead of snapping), and egg boxes or a 2 by 5 grid of cups for tactile ten-frames (lesson 1).
- Two sets of digit cards 0 to 9 per pair, so children can build and point to two-digit answers (every lesson).
- Story-problem cards for lesson 5 (see lesson 5), with a small picture on each card.
- Whiteboards and pens, or paper and pencils, for drawings and number sentences.

⚠ Safety and access

- Use cubes and counters too large to swallow; no rubber bands.
- Cube towers and the raised line on the mat let children who are blind or have low vision feel tens and ones; large-print or tactile hundred squares help too.
- Pointing to digit cards, moving cubes, sign, writing or a communication device all count as answers. Story problems are read aloud, so reading is not what is being checked.
- No public timing or races. "Quick" means without counting all from 1, in the child's own way.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week:

- "What is 8 + 7? How did you work it out?" (15; for example 7 + 7 + 1, or 8 + 2 + 5.) "What is 13 - 5?" (8; for example "5 and what make 13?")

- "What is 43 + 25?" (68: 4 tens and 2 tens are 6 tens, 3 ones and 5 ones are 8 ones.)
- "What is 37 + 48? Show me with cubes or a drawing." (85: 7 ones and 8 ones are 15 ones, which is 1 ten and 5 ones, so 3 + 4 + 1 = 8 tens and 5 ones.)
- "What is 62 - 27? How can you check?" (35: break a ten so there are 5 tens and 12 ones, take 2 tens and 7 ones, leaving 3 tens and 5 ones; check 35 + 27 = 62.)
- "56 leaves were on a branch. The wind blew some away. Now there are 29. How many blew away?" (27; 29 + □ = 56 or 56 - 29 = □.)
- "Duck had 18 pebbles. Duck found 20 more, then gave 9 to Hen. How many pebbles does Duck have now?" (29: 18 + 20 = 38, then 38 - 9 = 29.)

Record yes, with help or not yet for each part. A child who answers correctly by counting in ones has the answer but not yet the tens-and-ones method; note that rather than marking it wrong.

Sources

- Common Core State Standards, Grade 2 Operations and Algebraic Thinking (read to check what is usually expected in grade 2, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/OA/
    - 2.OA.A.1 one- and two-step word problems within 100 involving adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, using drawings and equations with a symbol for the unknown; 2.OA.B.2 fluently add and subtract within 20 using mental strategies, and by the end of grade 2 know from memory all sums of two one-digit numbers.
- Common Core, Grade 2 Number and Operations in Base Ten: https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/NBT/
    - 2.NBT.B.5 fluently add and subtract within 100 using strategies based on place value, properties of operations and the relationship between addition and subtraction; 2.NBT.B.6 add up to four two-digit numbers (lesson 3 stretch only); 2.NBT.B.9 explain why addition and subtraction strategies work, using place value (explanations may be supported by drawings or objects).
- Common Core, Grade 2 introduction (its critical areas include fluency with addition and subtraction within 100): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/introduction/

Known gaps and cautions

- Peer check: Gemini 4 Argon did a script-verified text check at 6e0e80a (keys, working, timings and links; as reported, not an educator or accessibility review) and reported 0 errors (see PEERCHECKS.md).
- Knowing all sums of two one-digit numbers from memory is an end-of-year aim in the source; this unit starts that work, and the daily quick-facts practice should continue all year.
- Odd and even numbers and adding equal groups (2.OA.C.3, 2.OA.C.4) are left for term 2, where the year plan puts equal groups. Three-digit numbers and adding within 1000 are also term 2.
- Problems with the unknown at the start ("Some stamps... now there are 60") are the hardest kind and appear in lesson 5 as a stretch task.
- A written column method is not taught. Children record tens and ones in their own way (drawings of towers and cubes, or splitting numbers into tens and ones); a written method can follow once the meaning is secure.

- The names in story problems (Remy, Duck, Hen, Mouse, Goat, Bear) are the made-up Little Hill characters used in other units. Swap in names and objects that suit your class, but avoid real children's belongings or families.
- The lesson order, timings and examples are my choice, not taken from a source.

Lesson 1: Facts within 20: checking and speeding up

Unit overview - Next: Lesson 2

About 45 minutes.

Goal

Children recall the grade 1 strategies for facts within 20 (doubles, near doubles, making ten, "what do I add?"), choose one for a given fact, and start answering more facts without counting all from 1.

Materials

- Two ten-frames and 20 counters per child, in two colours or two kinds of object.
- Fact cards per pair: 12 cards, each with one fact on it: 6 + 6, 7 + 8, 9 + 5, 8 + 6, 5 + 6, 9 + 9, 13 - 4, 15 - 8, 12 - 9, 17 - 8, 14 - 7, 11 - 3. Write the answer small on the back.
- Four strategy cards per pair, each with a word and a picture: double (two equal towers), near double (two towers, one cube taller), make ten (a full ten-frame), what do I add? (a question mark).
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Pairs to ten. Show 6 on a ten-frame. "How many more make 10?" (4.) Repeat with 3 (7), 8 (2), 1 (9) and 5 (5). Children answer with fingers, counters or digit cards.

✎ Main activity (32 min)

1. **Remember the strategies (10 min).** Work one fact of each kind on the board with ten-frames:
    - 7 + 7 = 14 (double).
    - 7 + 8 = 7 + 7 + 1 = 15 (near double).
    - 9 + 6: move 1 counter across to fill the first frame, so 10 + 5 = 15 (make ten).
    - 13 - 9: "9 and what make 13?" 9 + 1 = 10, then 3 more, so 4 (what do I add?).
2. **Sort the facts (12 min).** Pairs lay out the four strategy cards and put each fact card under the strategy they would use, then work it out and check the back. More than one place can be right (8 + 6 can be a near double, 7 + 7, or make ten, 8 + 2 + 4). Answers: 6 + 6 = 12, 7 + 8 = 15, 9 + 5 = 14, 8 + 6 = 14, 5 + 6 = 11, 9 + 9 = 18, 13 - 4 = 9, 15 - 8 = 7, 12 - 9 = 3, 17 - 8 = 9, 14 - 7 = 7, 11 - 3 = 8.
3. **Quick facts game (10 min).** Pairs shuffle the fact cards face down. One child turns a card and answers in their own way; the partner checks the back. If the answer took a

long count, the pair talks about which strategy would be quicker next time. Swap after each card. No timing and no scores.

✓ Discussion questions (with answers)

- Why is 7 + 8 easy if you know 7 + 7? 8 is one more than 7, so the answer is one more than 14: 15.
- For 12 - 9, why is "what do I add?" quicker than counting back 9? 9 is close to 12; adding up from 9 to 12 is just 3 steps.
- Can a fact belong to more than one strategy? Yes; any strategy that gives the right answer works.

✓ Closing check (8 min, with answers)

Each child answers on a whiteboard, with digit cards or by pointing: "7 + 8" (15), "14 - 6" (8; 6 and 8 make 14), "9 + 6" (15). Then one child at a time quietly tells the teacher which strategy they used for one of the three.

Vocabulary

double, near double, make ten, fact, strategy

Differentiation

- **Support:** stay with doubles and make ten for sums to 14, using ten-frames.
- **Stretch:** make a fact card of your own for each strategy, and explain why 6 + 8 can be solved as a double of 7 (take 1 from 8 and give it to 6: 7 + 7 = 14).
- Ten-frames can be made tactile (counters in an egg box or a 2 by 5 grid of cups). Fact cards can be read aloud by the partner or made in large print or braille.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Some children still count all from 1. That is fine in this lesson; note who does, and give them daily make-ten and doubles practice.
- "What do I add?" is a way to think about a take-away, not a different operation. Saying both ("13 take away 9 is 4, because 9 and 4 make 13") helps.

Lesson 2: Adding tens and ones

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 45 minutes.

Goal

Children add two two-digit numbers where the ones do not make a new ten, by adding tens with tens and ones with ones, using cubes, a tens-and-ones mat and the hundred square.

Materials

- Per pair: 10 towers of ten linking cubes, 20 loose cubes, a tens-and-ones mat and a hundred square.
- A large hundred square for the board.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

10 more and 10 less. Point to 47 on the big hundred square. "What is 10 more?" (57, the square below.) "10 less?" (37, the square above.) Repeat with 62 (72, 52) and 18 (28, 8).

✎ Main activity (30 min)

1. **Build both numbers (10 min).** On the board: 34 + 25. Build 34 (3 towers, 4 loose cubes) and 25 (2 towers, 5 loose) on the mat, towers on the tens side and loose cubes on the ones side. "Tens with tens: 3 tens and 2 tens are 5 tens. Ones with ones: 4 and 5 are 9. 5 tens and 9 ones is 59." Pairs build and add:
    - 42 + 36 (78)
    - 51 + 27 (78)
    - 23 + 45 (68)
2. **Hundred square jumps (10 min).** Show 34 + 25 on the big hundred square: start at 34, jump down 2 rows for 2 tens (44, 54), then 5 squares along for 5 ones (55, 56, 57, 58, 59). Pairs use their own squares for 23 + 45 and check it matches the cubes (68). Then 60 + 38 (98: start at 60, jump down 3 rows to 90, then 8 along to 98).
3. **Record it your own way (10 min).** Children write or draw one of their sums: drawings of towers and cubes, or splitting into tens and ones (34 + 25: 30 + 20 = 50, 4 + 5 = 9, 50 + 9 = 59). Any clear record counts.

✓ Discussion questions (with answers)

- Why do we put tens with tens and ones with ones? They are different sizes; 5 tens and 9 ones is 59, but adding 3 to 5 as if both were ones would give the wrong number.
- On the hundred square, why does jumping down one row add 10? Each row has ten squares, so the square below is always 10 more.
- Does 25 + 34 give the same answer as 34 + 25? Yes, 59; the order does not change the total.

✓ Closing check (10 min, with answers)

"Show me 32 + 46 with cubes or on the hundred square, then show the answer with digit cards." (78.) Then "14 + 53." (67.) Note who adds tens and ones and who counts every cube in ones.

Vocabulary

tens, ones, tower, loose cubes, hundred square, row

Differentiation

- **Support:** add a multiple of ten first (34 + 20 = 54), then a two-digit number with small ones (34 + 21 = 55).
- **Stretch:** find two different pairs of two-digit numbers that add to 79 without making a new ten (for example 45 + 34 and 23 + 56).
- Children who are blind or have low vision can feel the towers and loose cubes on either side of the raised line on the mat; the hundred square can be a large-print or tactile copy, or skipped in favour of cubes.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Some children add all four digits together (3 + 4 + 2 + 5 = 14). Building with towers and loose cubes shows why that does not work.
- Keep to sums where the ones total 9 or less in this lesson; lesson 3 deals with making a new ten.

Lesson 3: Adding when the ones make a new ten

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 45 minutes.

Goal

Children add two two-digit numbers where the ones make a new ten, by joining ten loose cubes into a tower, and explain why the tens go up by one.

Materials

- Per pair: 10 towers of ten linking cubes, 20 loose cubes and a tens-and-ones mat.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Make ten with loose cubes. Show 8 loose cubes and 5 loose cubes. "How many ones? Can we make a ten?" (13 ones: snap 10 into a tower, leaving 3, so 1 ten and 3 ones.) Repeat with 7 and 6 (13: 1 ten and 3 ones) and 9 and 9 (18: 1 ten and 8 ones).

✎ Main activity (30 min)

1. **A new ten (12 min).** On the board: 38 + 25. Build both numbers on the mat. "Tens: 3 and 2 are 5 tens. Ones: 8 and 5 are 13 ones. That is too many to leave loose: snap 10 of them into a tower and move it to the tens side. Now 6 tens and 3 ones: 63." Pairs build and add:
    - 47 + 26 (73)
    - 29 + 34 (63)
    - 56 + 18 (74)
2. **Say why (8 min).** Each pair explains one sum to another pair using the cubes: "7 ones and 6 ones are 13 ones. 10 ones make a new ten, so there are 4 + 2 + 1 = 7 tens and 3 ones: 73."
3. **Ones exactly ten (10 min).** 45 + 45. Ones: 5 and 5 are 10, which makes a whole new ten with no loose ones left. Tens: 4 + 4 + 1 = 9 tens, 0 ones:
4. Pairs try 36 + 24 (60) and 17 + 53 (70), then record one sum in their own way.

✓ Discussion questions (with answers)

- Why does the tens number go up by one in 38 + 25? The 13 ones contain a full ten, which joins the other tens.

- Why is 45 + 45 not "8 tens and 10 ones"? 10 ones is the same as 1 ten, so we write it as 9 tens and 0 ones: 90.
- How can you tell before adding whether the ones will make a new ten? If the two ones digits add to 10 or more.

✓ Closing check (10 min, with answers)

"Show me 27 + 35 with cubes or a drawing and tell me or a partner what happened to the ones." (62: 7 and 5 are 12 ones, a new ten and 2 ones, so 2 + 3 + 1 = 6 tens and 2 ones.) Then, with digit cards, "48 + 16." (64.)

Vocabulary

new ten, snap, too many ones, exchange

Differentiation

- **Support:** start with a two-digit number plus a one-digit number that makes a new ten (27 + 5 = 32, as in grade 1), then 27 + 15.
- **Stretch:** add four two-digit numbers by finding friendly pairs: 12 + 30 + 18 + 20 (12 + 18 = 30, then 30 + 30 + 20 = 80).
- Children who cannot snap cubes can slide ten loose cubes into a cup marked "ten" instead; an adult or partner can build while the child directs.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- A common error is writing 513 for 38 + 25 (5 tens, then 13 ones). Ask "Is this more than 100? Is that sensible?" and rebuild with cubes.
- "Exchange" is optional vocabulary; "snap ten ones into a ten" is enough.

Lesson 4: Taking away, breaking a ten, and checking by adding

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 45 minutes.

Goal

Children take away a two-digit number from a two-digit number, break a ten into ten ones when there are not enough ones, check by adding, and count up when the two numbers are close.

Materials

- Per pair: 10 towers of ten linking cubes, 20 loose cubes, a tens-and-ones mat and a hundred square.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Take away tens. "70 - 30?" (40.) "85 - 20?" (65.) "56 - 10?" (46.) Children use towers or the hundred square (jump up a row for each ten).

✎ Main activity (32 min)

1. **Enough ones (7 min).** 68 - 25: build 68, take away 2 towers and 5 loose cubes, leaving 4 towers and 3 cubes: 43. Pairs try 79 - 34 (45).
2. **Breaking a ten (12 min).** 52 - 17: build 52 (5 towers, 2 loose). "We need to take 7 ones but have only 2. Break one tower into 10 loose cubes: now 4 tens and 12 ones. Take 7 ones: 5 left. Take 1 ten: 3 tens left. 3 tens and 5 ones: 35." Pairs try:
    - 43 - 18 (25)
    - 60 - 24 (36: break a tower, so 5 tens and 10 ones; take 4 ones, leaving 6; take 2 tens, leaving 3)
3. **Check by adding (6 min).** "If 52 - 17 = 35, then 35 + 17 should be 52." Pairs put the 35 and the 17 back together with cubes (35 + 17 = 52, a new ten made from 5 and 7 ones), then check 25 + 18 = 43 and 36 + 24 = 60.
4. **Close numbers: count up (7 min).** 61 - 58: taking away 58 cubes is slow. "58 and what make 61?" On the hundred square: 58 to 60 is 2, 60 to 61 is 1, so 3. Pairs try 72 - 69 (3) and 40 - 37 (3).

✓ Discussion questions (with answers)

- Why do we break a ten in 52 - 17? There are only 2 ones and we need to take 7; one ten becomes 10 ones, giving 12.
- How does adding check a take-away? If we put back what we took away, we should get the number we started with.
- When is counting up quicker than taking away? When the two numbers are close together, like 61 and 58.

✓ Closing check (8 min, with answers)

"Show me 54 - 28 with cubes or a drawing, then check it by adding." (26: break a ten so there are 4 tens and 14 ones; take 8 ones, leaving 6; take 2 tens, leaving 2 tens; check 26 + 28 = 54.) Answer with digit cards, cubes, words or a drawing.

Vocabulary

take away, break a ten, check, count up, difference

Differentiation

- **Support:** stay with take-aways that need no breaking (68 - 25), then a one-digit number that needs breaking (32 - 5 = 27).
- **Stretch:** explain why 50 - 26 and 54 - 30 give the same answer (24 both times: adding 4 to both numbers keeps the difference the same).
- An adult or partner can break and move towers while the child directs; a child can answer by pointing to digit cards.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- The most common error is taking the smaller ones digit from the larger (52 - 17 giving 45, because "7 take away 2 is 5"). Building 52 and trying to take 7 loose cubes shows why that does not work.
- Keep "borrowing" out of the language; "break a ten into ten ones" says what happens.

Lesson 5: One-step and two-step story problems

Unit overview - Previous: Lesson 4

About 45 minutes.

Goal

Children solve one-step story problems within 100 of the five kinds and two-step story problems, by drawing the story and writing a number sentence with a box for the unknown.

Materials

- Story-problem cards (one set per pair; the problems are in the steps below), each with a small picture, read aloud by the teacher or a partner.
- Per pair: 10 towers of ten linking cubes, 20 loose cubes and a tens-and-ones mat.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Quick facts from lesson 1: "8 + 5?" (13.) "16 - 9?" (7.) "6 + 7?" (13.)

✎ Main activity (32 min)

1. **Draw the story (10 min).** Read: "Remy had 36 shells and found 27 more. How many shells now?" Draw a long bar split into two parts, 36 and 27, with a box above the whole bar. Write 36 + 27 = □. Solve with cubes or tens and ones (63). Then: "The Little Hill shop had 54 apples. Some were sold. Now there are 38. How many were sold?" Draw a bar for 54 split into 38 and a box. Write 38 + □ = 54 or 54 - 38 = □ (16).
2. **Problem cards (14 min).** Pairs read each card (or have it read to them), draw the story, write a number sentence with a box, and solve:
    - "45 boats are in the harbour. 28 are red and the rest are blue. How many are blue?" (17; 28 + □ = 45.)
    - "Hen has 62 seeds. Mouse has 25 fewer seeds than Hen. How many seeds does Mouse have?" (37; 62 - 25 = □.)
    - "Goat picked 71 apples and Duck picked 48. How many more did Goat pick?" (23; 48 + □ = 71.)
    - Stretch: "Bear had some stamps. Bear got 25 more. Now Bear has 60. How many stamps did Bear have at the start?" (35; □ + 25 = 60.)
3. **Two steps (8 min).** "Bear had 40 berries, ate 12, then picked 25 more. How many berries now?" Draw two steps: 40 - 12 = 28, then 28 + 25 = 53. Pairs try: "23 people

are on the Little Hill bus. 15 get on and 9 get off. How many are on the bus now?" (29: 23 + 15 = 38, then 38 - 9 = 29.)

✓ Discussion questions (with answers)

- In the apple problem, why can we add or take away? The 38 apples left and the apples sold make the 54 we started with; 38 + □ = 54 and 54 - 38 = □ find the same missing part.
- "Mouse has 25 fewer" has the word "fewer". Do we add or take away? Take away: Mouse has less than Hen, so 62 - 25.
- How do you know a problem has two steps? Two things happen, one after the other, and we need the first answer to do the second.

✓ Closing check (8 min, with answers)

Read aloud: "Duck had 34 pebbles and found 19 more. Then Duck gave 20 to Hen. How many pebbles does Duck have now?" Children draw the story and answer with digit cards, cubes, words or a drawing. (33: 34 + 19 = 53, then 53 - 20 = 33.)

Vocabulary

story problem, bar, part, whole, unknown, fewer, more than, two steps

Differentiation

- **Support:** one-step problems with numbers to 50 that do not make or break a ten; act the story out with cubes before drawing.
- **Stretch:** write a two-step story problem for a partner, with the answer on the back.
- Story cards can be in large print or braille, read aloud, or acted with objects; the bar can be built from cube towers instead of drawn. Answers can be given by pointing or with a communication device.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Key words can mislead: "more" appears in "How many more did Goat pick?", which is a take-away or missing-part problem. Drawing the story is more reliable than hunting for words.
- The bar drawing is one way to show a story; any drawing that shows the parts and the whole works.

Grade 2 Mathematics: Hundreds, Tens and Ones to 1000 (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 10 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard.
>
> Text-checked by one other agent (Gemini 4 Argon); see Known gaps.

Back to the grade 2 year plan - Term 2. Builds on grade 1: Tens and ones to 100 and grade 2 term 1: Adding and taking away within 100

Big idea

Ten ones make a ten, and ten tens make a hundred. The three digits of a number like 243 tell us how many hundreds, tens and ones there are, and a zero means "none of these". Because of this, we can compare numbers by looking at the hundreds first, and we can add or take away 10 or 100 by changing just one digit. Some numbers of things can be shared into pairs with none left over (even) and some cannot (odd), and things set out in equal rows can be counted by adding the rows.

Outcomes

By the end of the unit most children can:

1. Show that 100 is ten tens, and that 100, 200, 300 and so on up to 900 are one, two, three and so on up to nine hundreds, with no tens and no ones.
2. Build, read and write three-digit numbers as numerals, as number names and as hundreds, tens and ones (243 = 200 + 40 + 3), including numbers with a zero (706 is 7 hundreds, 0 tens and 6 ones).
3. Count on within 1000, including across a hundred (198, 199, 200, 201), and count in fives, tens and hundreds from different starting numbers.
4. Compare two three-digit numbers by looking at the hundreds, then the tens, then the ones, and record the result with >, = or <.
5. Say 10 more, 10 less, 100 more and 100 less than a number from 100 to 900 in their heads, and add or take away whole hundreds (300 + 400 = 700) or whole tens (570 - 40 = 530) when no new ten or hundred is made.
6. Tell whether a group of up to 20 objects is odd or even by pairing them, write an even number as a double (14 = 7 + 7), and find the total of an array of up to 5 rows and 5 columns by adding equal rows (4 + 4 + 4 = 12).

Prior knowledge

The grade 1 unit on tens and ones to 100 (tens and ones, comparing two-digit numbers with >, = and <, 10 more and 10 less) and the grade 2 term 1 unit on adding and taking away within 100. Children who are still building two-digit place value can work with hundreds and tens only (300, 340) before adding ones, in every lesson.

Lessons

Each lesson is about 45 minutes: a short whole-class start, paired work with cubes and hundred cards, and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so counting in fives, tens and hundreds should come back in short daily practice (5 minutes).

1. Ten tens make a hundred
2. Hundreds, tens and ones
3. Counting in fives, tens and hundreds, and comparing
4. 10 more, 10 less, 100 more, 100 less
5. Odd and even, and equal rows

Materials (whole class unless noted)

- Linking cubes too large to swallow, per pair: 10 towers of ten and 20 loose cubes (lessons 1 to 4), as in term 1. Straw bundles of ten wrapped with tape (no rubber bands, which may contain latex) work as well.
- Hundred cards, per pair: 9 squares of card, each the size of ten towers laid side by side, with a 10 by 10 grid drawn on and the grid lines raised with glued string or thick glue so they can be felt (lessons 1 to 4). Ten extra hundred cards for the class (lesson 1).
- A hundreds-tens-ones mat per pair: a sheet with three columns labelled hundreds, tens and ones, with raised lines (glued string) between them (lessons 2 to 4).
- Three sets of digit cards 0 to 9 per pair, so children can build and point to three-digit numbers (lessons 2 to 4).
- Number-name word cards per pair, in large print: one to nine, ten to nineteen, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, and hundred (lesson 2).
- Symbol cards >, = and < per pair, large enough to feel the shape (lessons 3 and 4).
- A large number line from 0 to 1000 marked in hundreds, for the board (lessons 3 and 4).
- A counting-in-tens chart for the board and one large-print copy per pair: ten rows, 10 to 100 in the first row, 110 to 200 in the second, and so on to 910 to 1000, with optional raised lines between rows (lesson 4).
- Hen's seed shop picture cards: a loose seed, a packet of 10 seeds and a box of 100 seeds (10 packets), plus order cards (lessons 1 and 2; see lesson 1).
- 20 counters per child, and a squared mat (a sheet ruled into 5 rows of 5 squares, with raised lines) per child (lesson 5).
- Whiteboards and pens, or paper and pencils.

⚠ Safety and access

- Use cubes and counters too large to swallow; no rubber bands.
- Towers, raised grid lines on the hundred cards and raised column lines on the mat let children who are blind or have low vision feel hundreds, tens and ones. Large-print or braille digit cards help children who read them.
- Pointing to digit cards, moving cubes and cards, sign, writing or a communication device all count as answers. Saying number names aloud is not required.
- No public timing or races. Counting games go round the class or pair, and a child may pass.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week:

- "Show me 100 with towers. How many tens is it?" (10.) "Show me 600 with hundred cards." (6 cards.)
- "Build 438. How many hundreds, tens and ones? Write it as hundreds, tens and ones." (4 hundreds, 3 tens, 8 ones; 400 + 30 + 8.)
- Show the digit cards 5, 0, 6. "Read this number and build it." (Five hundred six: 5 hundreds, 0 tens, 6 ones.)
- "Count on in tens from 370." (380, 390, 400, 410.) "Count in fives from 225 to 250." (230, 235, 240, 245, 250.)
- "Which is more, 614 or 641? Put the right sign between them." (641; 614 < 641, because 1 ten is less than 4 tens.)
- "What is 100 more than 752? 10 less than 752? What is 400 + 500?" (852; 742; 900.)
- "Is 13 odd or even? Show me with counters." (Odd: 6 pairs and 1 left over.) "Make 3 rows of 5 counters and write an adding sentence for the total." (5 + 5 + 5 = 15.)

Record yes, with help or not yet for each part. A child who counts every cube in ones has the answer but not yet the place-value idea; note that rather than marking it wrong.

Sources

- Common Core State Standards, Grade 2 Number and Operations in Base Ten (read to check what is usually expected in grade 2, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/NBT/
  - 2.NBT.A.1 the three digits of a three-digit number represent amounts of hundreds, tens and ones ("706 equals 7 hundreds, 0 tens, and 6 ones"), with 100 as "a bundle of ten tens" and 100, 200 ... 900 as one to nine hundreds; 2.NBT.A.2 count within 1000 and skip-count by 5s, 10s and 100s; 2.NBT.A.3 read and write numbers to 1000 using base-ten numerals, number names and expanded form; 2.NBT.A.4 compare two three-digit numbers using

    , = and <; 2.NBT.B.8 mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from it.

- Common Core, Grade 2 Operations and Algebraic Thinking: https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/OA/
  - 2.OA.C.3 odd or even for groups of up to 20 objects, "by pairing objects or counting them by 2s", and an even number as a sum of two equal addends; 2.OA.C.4 the total of a rectangular array with up to 5 rows and up to 5 columns, as a sum of equal addends.

Known gaps and cautions

- One other agent (Gemini 4 Argon, at 74bdc04) text-checked this unit on 2026-11-10 and reported no errors. That is a text check by an AI agent, not a teacher review or classroom test.
- Adding and taking away three-digit numbers where a new ten or hundred is made, or one has to be broken (2.NBT.B.7), is only touched as a stretch task (10 more than 395, 10 less

than 608). Adding up to four two-digit numbers (2.NBT.B.6) is not taught here. The grade 2 year plan does not give either a full unit, so a teacher may want to add one.

- English number names differ: some speakers say "five hundred six" and others "five hundred and six". Both are accepted; use the form your school uses on the word cards.
- Hundred cards are flat while towers are not, so a hundred card stands for ten towers rather than looking exactly like them. Lesson 1 has children lay towers on a card to check.
- Arrays stop at 5 by 5 and only use adding; multiplying is left for grade 3.
- Hen's seed shop and the other names (Duck, Mouse, Goat, Bear) are the made-up Little Hill characters used in other units.
- The lesson order, timings, examples and the seed-shop context are my choice, not taken from a source.

Lesson 1: Ten tens make a hundred

Unit overview - Next: Lesson 2

About 45 minutes.

Goal

Children see that ten tens make a hundred, use hundred cards to stand for a hundred, and count in hundreds to 1000.

Materials

- Per pair: 10 towers of ten linking cubes, 20 loose cubes and 9 hundred cards (10 by 10 grid, raised lines).
- Ten extra hundred cards for the class.
- Hen's seed shop picture cards: a loose seed, a packet of 10 seeds, and a box of 100 seeds (10 packets), plus order cards: "3 boxes", "5 boxes", "2 boxes and 4 packets", "6 boxes, 1 packet and 7 seeds".
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count in tens to 100 together, holding up a tower for each ten: 10, 20, 30 ... 100. "How many towers did we need to make 100?" (10.)

✎ Main activity (30 min)

1. **Ten towers on a card (10 min).** Each pair lays its 10 towers side by side on one hundred card. "The towers cover the whole card. Ten tens is one hundred." Children count the towers in tens to check (10, 20 ... 100) and run a finger along one row of the grid (10 squares) and down one column (10 rows). From now on, one hundred card stands for ten towers.
2. **Counting in hundreds (10 min).** Pairs put their hundred cards in a line along the floor or tables, and the class counts them in hundreds: 100, 200, 300 ... When the line has 9 cards, ask "What is one more hundred?" Add one of the class's extra cards: 1000, "one thousand", which is ten hundreds. Count back in hundreds from 1000 to 100. Ask "How many hundreds make 700?" (7) and "What number is 4 hundreds?" (400).
3. **Hen's seed shop (10 min).** Show the seed shop cards: Hen sells loose seeds, packets of 10 and boxes of 100. Read each order card; pairs build it with hundred cards, towers and cubes and say the number:
    - "3 boxes" (300)
    - "5 boxes" (500)
    - "2 boxes and 4 packets" (240)

    - "6 boxes, 1 packet and 7 seeds" (617)

✓ Discussion questions (with answers)

- How many ones are in a hundred? 100: ten tens, and each ten is ten ones.
- Why do we use a card instead of ten towers? It is quicker to build and count big numbers; the card covers exactly the same space as ten towers.
- What is ten hundreds? One thousand (1000).

✓ Closing check (10 min, with answers)

"Show me 400 with hundred cards." (4 cards.) "How many tens make 100?" (10.) "How many hundreds is 700?" (7.) Note who counts the cards in hundreds and who counts the grid squares one by one.

Vocabulary

hundred, hundred card, tens, ones, thousand, box, packet

Differentiation

- **Support:** build 100 with towers first, then 200 with towers on two cards, before using cards alone.
- **Stretch:** "Hen has 1000 seeds in boxes. How many boxes?" (10.) "How many packets of 10 make 300?" (30.)
- Children who are blind or have low vision feel the towers on the card and the raised grid; the line of cards can be on a table so it can be touched along its length.

⚠ Safety

Use cubes too large to swallow. Keep the floor line of cards out of walkways.

Teacher notes and misconceptions

- Some children think a hundred card is "one" because it is one piece. Keep asking "How many tens is that card?" and let them check with towers.
- Thousand is mentioned only as ten hundreds; numbers above 1000 are not taught.

Lesson 2: Hundreds, tens and ones

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 45 minutes.

Goal

Children build, read and write three-digit numbers as numerals, number names and hundreds, tens and ones, including numbers with a zero.

Materials

- Per pair: 9 hundred cards, 10 towers of ten, 20 loose cubes and a hundreds-tens-ones mat.
- Three sets of digit cards 0 to 9 per pair.
- Number-name word cards per pair (one to nine, ten to nineteen, twenty to ninety, hundred).
- Hen's seed shop picture cards (from lesson 1).
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count in hundreds from 100 to 900 and back, pointing to hundred cards. Show the seed shop box card: "How many seeds in 3 boxes?" (300.)

✎ Main activity (30 min)

1. **Build it and read it (10 min).** On the board: 243. Build it on the mat: 2 hundred cards, 4 towers and 3 loose cubes. "2 hundreds, 4 tens and 3 ones: two hundred forty-three." Show the digit cards 2, 4, 3 under the right columns. Pairs build, put digit cards under and read:
    - 365
    - 512
    - 128
2. **When there is a zero (10 min).** Build 706: 7 hundred cards, no towers, 6 cubes. "The tens column is empty, so we write 0 tens: seven hundred six." Then build 760 and 67 next to it. "Which has the most? Which digit tells you?" (760: it has 7 hundreds and 6 tens; 67 has no hundreds.) Pairs build 607 and 670 and say how they differ (607 has 0 tens and 7 ones; 670 has 7 tens and 0 ones).
3. **Three ways to write it (10 min).** Show 352 three ways: the numeral, the name (three hundred fifty-two, with word cards) and hundreds, tens and ones (300 + 50 + 2). Pairs choose two numbers they built and write each in all three ways. Words may be built from word cards instead of written.

✓ Discussion questions (with answers)

- Why can't we leave out the 0 in 706? Without it, the 7 and 6 move up and the number reads 76, which is much less.
- In 352, what is the 5 worth? 50, or 5 tens.
- Is 300 + 50 + 2 the same number as 352? Yes, it is the same number split into hundreds, tens and ones.

✓ Closing check (10 min, with answers)

"Build 409 and tell me how many hundreds, tens and ones." (4 hundreds, 0 tens, 9 ones; 400 + 9.) "What number is 300 + 50 + 2?" (352.) Note who puts a 0 in the tens place and who writes 49.

Vocabulary

hundreds, tens, ones, digit, zero, number name

Differentiation

- **Support:** build only hundreds and tens first (340, 520), then add ones.
- **Stretch:** "Make the biggest and the smallest number you can with the digit cards 3, 0 and 8." (830 and 308; a number does not start with 0.)
- Children who are blind or have low vision feel the raised columns on the mat and the grid on the hundred cards; number names can be said, signed or pointed to instead of written.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Some children write two hundred forty-three as 20043, writing each part as they hear it. Building on the mat and putting one digit card under each column helps.
- Some speakers say "two hundred and forty-three". Both forms are fine.

Lesson 3: Counting in fives, tens and hundreds, and comparing

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 45 minutes.

Goal

Children count on within 1000, including across a hundred, count in fives, tens and hundreds from different starting numbers, and compare three-digit numbers using >, = and <.

Materials

- Per pair: 9 hundred cards, 10 towers of ten, 20 loose cubes, a hundreds-tens-ones mat, three sets of digit cards 0 to 9, and symbol cards

  , = and <.

- The large number line from 0 to 1000 marked in hundreds, for the board.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count in fives from 0 to 50 together, clapping on each number. Then count on in ones from 196 to 204, pointing near 200 on the number line: 197, 198, 199, 200, 201 ... "What happened after 199?" (A new hundred: 200.)

✎ Main activity (30 min)

1. **Counting from different places (10 min).** Go round the class or pair, each child saying the next number (a child may pass):
   - fives from 115 to 150 (120, 125, 130, 135, 140, 145, 150)
   - tens from 340 to 420 (350, 360, 370, 380, 390, 400, 410, 420)
   - hundreds from 50 to 950 (150, 250, 350 ... 950) Point to roughly where each count is on the number line.
2. **Which is more? (10 min).** Duck's sack has 352 seeds; Mouse's sack has
3. Pairs build both. "Look at the hundreds first: both have 3. Then the tens: 5 tens is more than 2 tens. So 352 is more." Write 352 > 325 and read it: "352 is greater than 325." Show that < means "is less than", and that the wide-open side of the sign faces the bigger number. Pairs compare and put a symbol card between:
   - 418 and 481 (418 < 481)
   - 609 and 690 (609 < 690)
   - 700 and 699 (700 > 699)
   - 534 and 534 (534 = 534)

4. **Make and compare (10 min).** Each child in a pair draws three digit cards and makes a three-digit number (not starting with 0). Together they build both, put the right symbol between and read the sentence. Play several rounds; there is no winner, only the sentence.

✓ Discussion questions (with answers)

- Why do we look at the hundreds first? A hundred is more than any number of tens and ones we could have in the other columns (99 at most), so the hundreds decide unless they are the same.
- 700 has only zeros after the 7. Why is it more than 699? 700 has 7 hundreds; 699 has only 6 hundreds and 9 tens and 9 ones, which is one less.
- What comes after 399? 400.

✓ Closing check (10 min, with answers)

"Which is more, 263 or 236? How do you know?" (263: same hundreds, and 6 tens is more than 3 tens.) "Put the right sign: 507 and 570." (507 < 570.) "Count on in tens from 280." (290, 300, 310.)

Vocabulary

greater than, less than, equal to, compare, count on, fives

Differentiation

- **Support:** compare numbers with different hundreds first (245 and 512), building both, before numbers with the same hundreds.
- **Stretch:** "Put these in order from least to greatest: 489, 498, 849, 894." (489, 498, 849, 894.) Count back in tens from 430 to 370 (420, 410, 400, 390, 380, 370).
- Children who are blind or have low vision feel the symbol cards and the built numbers; counts can be tapped on a table as well as said.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Some children say 418 is more than 481 "because 8 is bigger than 1". Going column by column, starting with the hundreds, fixes this.
- Counting on across a hundred (what comes after 399, or after 390 when counting in tens) is often shaky; keep it in daily counting practice.

Lesson 4: 10 more, 10 less, 100 more, 100 less

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 45 minutes.

Goal

Children say 10 more, 10 less, 100 more and 100 less than a number from 100 to 900 in their heads, and add or take away whole hundreds or whole tens when no new ten or hundred is made.

Materials

- Per pair: 9 hundred cards, 10 towers of ten, 20 loose cubes, a hundreds-tens-ones mat, three sets of digit cards 0 to 9 and symbol cards

  , = and <.

- The counting-in-tens chart for the board and one large-print copy per pair.
- The large number line from 0 to 1000 marked in hundreds.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Quick 10 more and 10 less within 100, as in term 1: "10 more than 47?" (57.) "10 less than 62?" (52.) "10 more than 90?" (100.)

✎ Main activity (30 min)

1. **Change one card (10 min).** Pairs build 346. "Add one hundred card. What number is it now?" (446.) "Take it away again and add one tower." (356.) "Which digit changed each time?" (The hundreds digit for 100 more, the tens digit for 10 more.) Pairs build, change and say:
   - 100 more than 521 (621)
   - 100 less than 874 (774)
   - 10 more than 235 (245)
   - 10 less than 653 (643)
2. **The counting-in-tens chart (10 min).** On the chart, 10 more is one step along a row and 100 more is one row down. Start at 340: one step along (350), one row down (450). Pairs find on their copies: 100 less than 580 (480), 10 more than 720 (730), 100 more than 260 (360). Then without the chart: "What is 100 more than 263?" (363.)
3. **Whole hundreds and whole tens (10 min).** "3 hundreds and 4 hundreds is 7 hundreds, so 300 + 400 = 700." Pairs work out in their heads and check with cards:
   - 300 + 400 (700)

- 800 - 500 (300)
- 240 + 30 (270)
- 570 - 40 (530)
- 600 + 200 + 100 (900)

✓ Discussion questions (with answers)

- Why does only one digit change when we add 100 to 346? We add one hundred, so only the number of hundreds changes; the tens and ones stay the same.
- Why is 300 + 400 like 3 + 4? We are adding hundreds instead of ones: 3 hundreds and 4 hundreds are 7 hundreds.
- On the chart, why is one row down 100 more? Each row has ten steps of 10, which is 100.

✓ Closing check (10 min, with answers)

"What is 100 more than 263?" (363.) "10 less than 263?" (253.) "What is 500 + 300?" (800.) "700 - 200?" (500.) Note who answers in their head and who needs to build.

Vocabulary

10 more, 10 less, 100 more, 100 less, in my head

Differentiation

- **Support:** keep building every number and changing one card or tower before trying in their heads.
- **Stretch:** "10 more than 395?" (405: 9 tens and 1 more ten make a new hundred.) "10 less than 608?" (598: a hundred is broken into ten tens.) Children check with cards and towers.
- Children who are blind or have low vision use the mat and cards; the counting-in-tens chart can have raised lines between rows, or be skipped in favour of building.

⚠ Safety

Use cubes too large to swallow.

Teacher notes and misconceptions

- Some children add 100 by adding 1 to the ones digit (346 becomes 347). Ask "What did you add: a cube, a tower or a hundred card?"
- The stretch tasks cross a hundred, which needs a new hundred to be made or a hundred to be broken. That is not expected of everyone in this unit.

Lesson 5: Odd and even, and equal rows

Unit overview - Previous: Lesson 4

About 45 minutes.

Goal

Children tell whether a group of up to 20 objects is odd or even by pairing, write an even number as a double, and find the total of an array with up to 5 rows and 5 columns by adding equal rows.

Materials

- 20 counters per child, too large to swallow.
- A squared mat per child: a sheet ruled into 5 rows of 5 squares, with raised lines.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count in twos to 20 together, then back to 0.

✎ Main activity (30 min)

1. **Can every counter have a partner? (12 min).** Take 12 counters and put them in pairs: 6 pairs and none left over. "12 is even." Take 13: 6 pairs and 1 left over. "13 is odd." Children test these and sort them into odd and even:
    - 8 (even: 4 pairs)
    - 11 (odd: 5 pairs and 1 left)
    - 14 (even: 7 pairs)
    - 17 (odd: 8 pairs and 1 left)
    - 20 (even: 10 pairs)
    - 9 (odd: 4 pairs and 1 left) Then split each even number into two equal rows and write it as a double: 12 = 6 + 6, 14 = 7 + 7, 20 = 10 + 10.
2. **Goat's carrot rows (13 min).** Goat plants carrots in rows with the same number in each row. Show 3 rows of 4 on the mat: "4 + 4 + 4 = 12." Children build each array on their mat with counters and write an adding sentence:
    - 2 rows of 5 (5 + 5 = 10)
    - 4 rows of 3 (3 + 3 + 3 + 3 = 12)
    - 3 rows of 5 (5 + 5 + 5 = 15)
    - 5 rows of 5 (5 + 5 + 5 + 5 + 5 = 25)
3. **Turn it round (5 min).** Turn the mat a quarter turn so 4 rows of 3 become 3 rows of 4. "Is it the same number of carrots?" (Yes, 12: 3 + 3 + 3 + 3 = 12 and 4 + 4 + 4 = 12.)

✓ Discussion questions (with answers)

- How can you tell 17 is odd without counting by ones? Put the counters in pairs: one is always left over.
- Is every double an even number? Yes: two equal rows can always be put together in pairs, one from each row.
- Why is adding rows quicker than counting every carrot? Each row has the same number, so we can count on in that number.

✓ Closing check (10 min, with answers)

"Is 15 odd or even? Show me." (Odd: 7 pairs and 1 left over.) "Write 16 as a double." (8 + 8.) "Make 2 rows of 4 and write an adding sentence." (4 + 4 = 8.)

Vocabulary

odd, even, pair, double, row, column, array

Differentiation

- **Support:** work with numbers up to 10 for odd and even, and arrays with 2 rows first.
- **Stretch:** "Make every array you can with 12 counters, using no more than 5 rows and 5 columns." (3 rows of 4 and 4 rows of 3.) "Is the total of two odd numbers odd or even? Try some." (Even: the two left-over counters make a pair.)
- Children who are blind or have low vision feel the raised squares on the mat and the pairs of counters; answers can be said, signed or shown with counters.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Some children think a number is even if it "looks big" or ends in a big digit. Pairing real counters is the check.
- Multiplying and the times sign are not taught here; arrays are counted by adding equal rows.

Grade 2 Mathematics: Measuring, Time, Money and Shapes (5 lessons, about 5 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 11 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard. Check coins, units and clock conventions against your school's rules and your country's money.
>
> Text-checked by two other agents (Gemini 3.8 Flash and Gemini 4 Argon); see Known gaps.

Back to the grade 2 year plan - Term 3. Builds on grade 1: Measuring length and composing shapes and grade 2 term 2: Hundreds, tens and ones to 1000

Big idea

We measure by counting equal units, and a ruler is a line of equal units already counted for us, starting from 0. A smaller unit needs more of them to cover the same length. Clocks and coins also use counting: the minute hand moves in fives, and coins of different values can be counted on from the biggest. Shapes can be named by counting their sides and corners, and a bar graph lets us count and compare a set of data at a glance.

Outcomes

By the end of the unit most children can:

1. Measure a length to the nearest whole centimetre with a ruler, starting at the 0 mark, and say how much longer one length is than another (25 cm is 10 cm longer than 15 cm).
2. Measure the same length in centimetres and in inches, explain why the number of centimetres is bigger (a centimetre is a smaller unit, so more of them are needed), and estimate lengths in centimetres, inches, feet or metres before measuring.
3. Tell and write the time to five minutes on a clock with hands and as digital time (4:25), and say whether a time in a story is a.m. or p.m.
4. Find the value of a set of coins (pennies, nickels, dimes and quarters) by counting on from the biggest value, write it with the ¢ sign, know that $1 is 100¢, and solve one-step story problems with made-up prices.
5. Name triangles, quadrilaterals, pentagons and hexagons by counting sides and corners, say how many faces a cube has, and put shapes together to make a new shape.
6. Make a bar graph with up to four categories, one square for each item, and use it to answer "how many altogether?" and "how many more?" questions.

Prior knowledge

The grade 1 unit on measuring length and composing shapes (lining up ends, measuring with equal units laid end to end, triangles, squares and rectangles, putting shapes together), counting in fives and tens from grade 2 term 2, and adding and taking away within 100 from grade 2 term 1. No earlier unit in this set teaches clocks or coins, so lesson 3 starts with o'clock and half past and lesson 4 starts by meeting the coins.

Lessons

Each lesson is about 45 minutes: a short whole-class start, paired work and a closing check. The five topics are quite different, so a teacher may want to spread each lesson over two or three days; the unit runs over about 5 weeks, with short daily practice (5 minutes) reading the class clock and counting coins.

1. Measuring with a centimetre ruler
2. Centimetres and inches
3. Telling the time to five minutes
4. Counting coins at the Little Hill market
5. Sides, corners and a shape bar graph

Materials (whole class unless noted)

- A 30 cm ruler per pair, marked in whole centimetres with the 0 clearly marked, and with inches on the other edge (lessons 1 and 2). Tactile rulers with raised or notched marks for children who need them.
- Paper strips per pair, cut exactly to length, each with its letter in large print and a raised dot sticker on the left end: A 10 cm, B 15 cm, C 20 cm, D 25 cm and E 12 cm (lessons 1 and 2). A few spare raised dot stickers.
- Linking cubes too large to swallow, about 10 per pair, as in grade 1 (lesson 1).
- Small classroom objects per pair to measure: a pencil, a crayon, a glue stick and a book (lessons 1 and 2).
- A large board ruler or number line from 0 to 40 (lesson 1).
- A metre stick and a 12-inch ruler (one foot) for the teacher (lesson 2).
- A two-column chart headed "centimetres" and "inches" for the board, and a copy per pair (lesson 2).
- A teaching clock with geared hands for the board, and a card clock per pair with an hour hand and a minute hand fixed by a paper fastener (lesson 3; see Safety and access).
- Digital time cards in large print, and Little Hill day cards (lesson 3; see lesson 3).
- A minutes card per pair for support: a ring of 5, 10, 15 ... 60 to sit around the card clock (lesson 3).
- Card coins per pair, too large to swallow, printed on both sides with the value raised (glued string or thick glue) so it can be felt: 4 quarters, 6 dimes, 6 nickels and 10 pennies, plus one card dollar bill (lesson 4).
- Little Hill market price cards with made-up prices (lesson 4; see lesson 4).
- Card shapes per pair in different sizes and turned different ways: triangles, quadrilaterals (including squares, rectangles and some with no equal sides), pentagons and hexagons (lesson 5).
- Pattern blocks too large to swallow, per pair: 6 triangles, 2 trapezoids, 2 rhombuses (diamonds) and 1 hexagon (lesson 5).
- A cube per pair, such as a large wooden or foam block (lesson 5).
- A shape bag per pair holding exactly 19 card shapes: 5 triangles, 7 quadrilaterals, 3 pentagons and 4 hexagons (lesson 5).
- Squared paper (2 cm squares) and dot paper per child; card squares and glue for raised bar graphs (lesson 5). Small stickers for marking corners (lesson 5).
- A bar graph for the board made with raised card squares: boats on the Little Hill pond, with sailing boats 3, rowing boats 5 and canoes 2 (lesson 5).
- Whiteboards and pens, or paper and pencils.

⚠ Safety and access

- **Small parts:** card coins, not real coins (real coins are a choking risk and are handled by many people). Pattern blocks and shapes too large to swallow.
- **Card clocks:** an adult fixes the paper fasteners before the lesson and tapes over the sharp ends on the back.
- **Seeing and touching:** tactile rulers, raised dot stickers on the strips, card coins with raised values, raised clock numbers (glued string or thick glue dots) and bar graphs made from stuck-on card squares let children who are blind or have low vision measure, read and compare by touch. Large-print or braille labels help children who read them. Children can feel the clock hands on the teaching clock or their card clock.
- **Answering:** pointing, moving cards, sign, writing or a communication device all count as answers. Saying number names aloud is not required.
- **Privacy:** times, prices and data in the lessons are about the made-up Little Hill characters and a bag of shapes. Never ask children about their own wake or bed times, family money, pocket money or what things cost at home.
- No public timing or races. Games go round the pair or group, and a child may pass.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the last week:

- Give strip E and a ruler. "How long is this strip?" (12 cm.) "Strip D is 25 cm. How much longer is it than strip E?" (13 cm.)
- "Measure strip C in centimetres and in inches." (20 cm; about 8 inches.) "Why is the number of inches smaller?" (An inch is longer than a centimetre, so fewer inches fit.)
- Set the teaching clock to 9:50. "What time is it? Write it." (9:50; ten to ten is also fine.) "Duck eats breakfast at 7:20. Is that a.m. or p.m.?" (a.m.)
- Give a quarter, a dime, a nickel and two pennies. "How much money is this?" (42¢.) "A ribbon costs 20¢ and a pencil costs 15¢. How much for both?" (35¢.)
- Show a card shape with 6 sides that are not all the same length. "What is this shape called? How do you know?" (A hexagon: 6 sides and 6 corners.) "How many faces does a cube have?" (6.)
- Show a bar graph of kites at the Little Hill fair: red 4, blue 6, yellow 2, green 3. "How many kites altogether?" (15.) "How many more blue kites than yellow kites?" (4.)

Record yes, with help or not yet for each part. A child who starts a ruler at the 1 mark and is 1 out every time has a measuring habit to fix, not a counting problem; note that rather than marking it wrong.

Sources

- Common Core State Standards, Grade 2 Measurement and Data (read to check what is usually expected in grade 2, not to claim alignment): https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/MD/
  - 2.MD.A.1 "Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes."; 2.MD.A.2 measure an object twice with units of different lengths and "describe how the two measurements relate to the size of the unit chosen"; 2.MD.A.3 "Estimate lengths using units of

inches, feet, centimeters, and meters."; 2.MD.A.4 measure "how much longer one object is than another".
    - 2.MD.B.5 adding and taking away within 100 in word problems about lengths, "by using drawings (such as drawings of rulers)".
    - 2.MD.C.7 "Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m."; 2.MD.C.8 "Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately." with the example "If you have 2 dimes and 3 pennies, how many cents do you have?"
    - 2.MD.D.10 "Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories."
- Common Core, Grade 2 Geometry: https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/2/G/
    - 2.G.A.1 "Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces." and "Identify triangles, quadrilaterals, pentagons, hexagons, and cubes."
- Common Core, Grade 1 Measurement and Data, read to check what children may have met before: 1.MD.B.3 "Tell and write time in hours and half-hours using analog and digital clocks." https://web.archive.org/web/2025/https://www.thecorestandards.org/Math/Content/1/MD/

Known gaps and cautions

- Two other agents (Gemini 3.8 Flash and Gemini 4 Argon, both at 122c91c) text-checked this unit on 2026-11-11 and reported no errors. Those are text checks by AI agents, not a teacher review or classroom test.
- Some grade 2 standards are not taught here: line plots of measurement data (2.MD.D.9), partitioning a rectangle into rows and columns of squares (2.G.A.2), and halves, thirds and fourths of circles and rectangles (2.G.A.3). Number lines (2.MD.B.6) appear only in lesson 1. A teacher may want to add these.
- The coins are United States coins because the Common Core standard names them. Schools in other countries should use their own coins and change the values in lesson 4. Writing amounts with a decimal point ($0.42) is not taught; amounts under a dollar are written with ¢.
- Inches and feet are used in the United States; many other countries use only centimetres and metres. Lesson 2 can be taught with centimetres and a longer unit such as a 5 cm card strip instead of inches.
- The strips in lessons 1 and 2 were chosen so the inch measurements are close to whole numbers (25 cm is about 9.8 inches, so "about 10 inches"). Children measure to the nearest whole unit, so small differences between pairs are expected.
- Many rulers have a gap before the 0 mark. Lesson 1 teaches starting at 0 for this reason.
- Clocks with hands are less common in some homes than digital ones, so some children will need much more practice than one lesson; the daily 5 minutes matters. Times are always on a five-minute mark; reading to the nearest five minutes when the hand is between marks is left for later practice.
- The prices, times and graph data are made up for Little Hill and are not real prices or schedules. Duck, Hen, Mouse, Goat and Mayor Hen are the made-up Little Hill characters used in other units.
- The lesson order, timings, strip lengths, examples and contexts are my choice, not taken from a source.

Lesson 1: Measuring with a centimetre ruler

Unit overview - Next: Lesson 2

Back to the grade 2 year plan

About 45 minutes.

Goal

Children measure lengths to the nearest whole centimetre with a ruler, starting at the 0 mark, and find how much longer one length is than another.

Materials

- A 30 cm ruler per pair, with the 0 clearly marked. Tactile rulers with raised or notched marks for children who need them.
- Paper strips per pair, cut exactly to length, each with its letter in large print and a raised dot sticker on the left end: A 10 cm, B 15 cm, C 20 cm, D 25 cm and E 12 cm. A few spare raised dot stickers.
- Small classroom objects per pair: a pencil, a crayon, a glue stick and a book.
- A large board ruler or number line from 0 to 40.
- Linking cubes too large to swallow, about 10 per pair, for the opening.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Hold up strip C next to a row of linking cubes laid end to end. "In grade 1 we measured by laying equal units with no gaps or overlaps. How many cubes long is this strip?" (Count with the class; the answer depends on your cubes.) "Different cubes give different answers. A ruler uses a unit everyone agrees on: the centimetre. The small spaces between the marks are each one centimetre, and the numbers have already counted them for us."

✎ Main activity (30 min)

1. **Start at 0 (10 min).** On the board ruler, show that the numbers count the spaces, not the lines: the space from 0 to 1 is the first centimetre. Show strip A lined up with the very end of a real ruler (many rulers have a gap before the 0) and read where it ends. Then line it up with the 0 mark. "Which way gives the right length?" (Starting at 0: strip A is 10 cm.) Next lay strip A from the 1 mark to the 11 mark. "It ends at 11. Is it 11 cm long?" (No: count the spaces from 1 to 11; there are 10.) Children who use a tactile ruler feel for the 0 notch first.
2. **Measure the strips (10 min).** Pairs measure strips A to D, taking turns to hold the ruler and to read the number, and write each length with "cm": A 10 cm, B 15 cm, C 20 cm, D 25 cm. Then they measure the pencil, crayon, glue stick and book to the nearest

whole centimetre. "If the end is between two numbers, which number is it closer to?" Answers for the objects will differ between pairs.

3. **How much longer? (10 min).** "Put strips D and B side by side, lined up at one end. How much longer is D?" Measure the part of D that sticks out, or take away: 25 - 15 = 10, so D is 10 cm longer. Repeat with C and A (10 cm) and D and A (15 cm). Then a story, drawn as a ruler on the board with jumps along the number line: "Goat's kite tail is 34 cm long. Duck's kite tail is 20 cm long. How much longer is Goat's?" (34 - 20 = 14, so 14 cm.) Show the jump from 20 to 34 on the board number line.

✓ Discussion questions (with answers)

- Why do we start at 0 and not at the end of the ruler? The numbers count centimetres from the 0 mark. Many rulers have a gap before the 0, so starting at the end gives the wrong length.
- Why is everyone's strip D the same length but their pencils different? The strips were cut to the same length; the pencils are different pencils.
- Can you measure a strip that does not start at 0? Yes, by counting the centimetres from where it starts to where it ends, but starting at 0 is easier.

✓ Closing check (10 min, with answers)

"Measure strip E." (12 cm.) "A strip starts at 2 and ends at 9 on a ruler. How long is it?" (7 cm.) "Mouse's ribbon is 18 cm long and Hen's ribbon is 30 cm long. How much longer is Hen's ribbon?" (12 cm.) Note who starts at 0 every time and who still starts at the end or at 1.

Vocabulary

centimetre, cm, ruler, length, measure, longer, shorter, how much longer, nearest

Differentiation

- **Support:** measure only strips A to D, which end on numbers that end in 0 or 5, before measuring objects. Put a spare raised dot sticker on the 0 mark of the ruler.
- **Stretch:** "Which two strips together are as long as strip D?" (A and B: 10 + 15 = 25.) "How much longer is strip D than strip E?" (13 cm.)
- Children who are blind or have low vision use a tactile ruler and the raised dot on the left end of each strip, and count notches from the 0 notch with a finger.

⚠ Safety

Use linking cubes too large to swallow. Rulers stay on tables, not waved.

Teacher notes and misconceptions

- Many children start at the end of the ruler or at 1. Ask "Where does the first centimetre start?" and let them check by counting spaces.

- Some children count the lines instead of the spaces and get one more than the length. Point to each space as it is counted.
- "Cm" is the short way to write centimetres; the "s" is never added.

Lesson 2: Centimetres and inches

Unit overview - Previous: Lesson 1 - Next: Lesson 3

Back to the grade 2 year plan

About 45 minutes.

Goal

Children measure the same strips in centimetres and in inches, explain why the number of centimetres is bigger, and estimate lengths before measuring.

Materials

- A 30 cm ruler per pair with inches on the other edge. Tactile rulers with raised or notched marks for children who need them.
- Paper strips A to E per pair from lesson 1 (A 10 cm, B 15 cm, C 20 cm, D 25 cm, E 12 cm), each with a raised dot sticker on the left end.
- Small classroom objects per pair: a pencil, a crayon, a glue stick and a book.
- A metre stick and a 12-inch ruler (one foot) for the teacher.
- A two-column chart for the board headed "centimetres" and "inches", and a copy per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

"Last time strip D was 25 cm long. Turn your ruler over. The other edge has bigger spaces. Each of those spaces is one inch." Show one centimetre and one inch on the board side by side. "Which is longer, a centimetre or an inch?" (An inch.)

✎ Main activity (30 min)

1. **Measuring in inches (10 min).** Pairs measure strips A to D on the inch edge, starting at 0, to the nearest whole inch, and write the answers in the inches column of their chart. The strips do not end exactly on an inch mark, so the class agrees which mark each end is closest to: A about 4 inches, B about 6 inches, C about 8 inches, D about 10 inches.
2. **Same strip, two numbers (10 min).** Pairs copy the centimetre lengths from lesson 1 into the other column: A 10, B 15, C 20, D 25. "Strip D is 25 cm and about 10 inches. It is the same strip. Why is the number of centimetres bigger?" Take ideas, then show with strip D: lay a finger on each centimetre space and then on each inch space. "An inch is longer than a centimetre, so fewer inches fit along the strip. A smaller unit needs more of them." Link to grade 1: when the class measured with smaller units, it needed more of them to cover the same length.

3. **Estimate, then measure (10 min).** Show the metre stick (100 cm) and the 12-inch ruler ("12 inches is one foot"; a 30 cm ruler is about one foot long). For each object, pairs first write a guess, then measure:
    - the pencil and the book in centimetres;
    - the crayon and the glue stick in inches;
    - with the teacher: "Is the classroom door wider or narrower than one metre?" The teacher holds the metre stick against the door so the class can check.

Guesses that are close count as good estimates; the point is to have a picture of each unit in mind.

✓ Discussion questions (with answers)

- If you measure the same book in centimetres and in inches, which number is bigger? The centimetres, because a centimetre is a smaller unit.
- Why did different pairs get "about 6 inches" for strip B when it does not end exactly on 6? It is closer to the 6 mark than to the 5 or 7 mark, so we say about 6 inches.
- Would you measure the classroom in centimetres or in metres? Metres would be quicker; centimetres would give a very big number.

✓ Closing check (10 min, with answers)

"Measure strip E in inches." (About 5 inches.) "A strip is 8 inches long. Will its length in centimetres be a bigger number or a smaller number?" (Bigger.) "Which do you need more of to measure the same book, centimetres or inches?" (Centimetres.) Note who explains with the size of the unit and who only says "because the numbers are different".

Vocabulary

inch, foot, metre, centimetre, estimate, about, unit, closest

Differentiation

- **Support:** measure strips C and D only, with a finger walking along each inch space while counting.
- **Stretch:** "Strip D is about 10 inches. Two strips D laid end to end would be about how many inches?" (About 20.) "How many centimetres?" (50.)
- Children who are blind or have low vision feel the bigger notches on the inch edge of a tactile ruler and compare one centimetre space with one inch space by touch.

⚠ Safety

Only the teacher handles the metre stick. Rulers stay on tables, not waved.

Teacher notes and misconceptions

- Some children think a bigger number means a longer object even when the units differ. Keep putting the same strip next to both edges of the ruler.
- On some rulers the inch edge is split into halves and quarters as well. Show children which marks are the whole inches (the longest marks with numbers).
- In countries that do not use inches, the same idea can be shown with centimetres and a 5 cm card strip as a longer unit.

Lesson 3: Telling the time to five minutes

Unit overview - Previous: Lesson 2 - Next: Lesson 4

Back to the grade 2 year plan

About 45 minutes.

Goal

Children read and set times to five minutes on a clock with hands, write them as digital times, and decide whether a time in a story is a.m. or p.m.

Materials

- A teaching clock with geared hands for the board, with raised numbers (glued string or thick glue dots) so it can be felt.
- A card clock per pair with an hour hand and a minute hand fixed by a paper fastener. An adult fixes the fasteners before the lesson and tapes over the sharp ends on the back.
- Digital time cards in large print: 3:00, 3:30, 4:25, 7:40, 10:05, 11:55.
- Little Hill day cards, each with a picture, a sentence and a time: "Hen opens the bakery at 7:15 in the morning", "The bus leaves for school at 8:40 in the morning", "Lunch is ready at the market at 12:10 in the afternoon", "Mayor Hen rings the town bell at 3:25 in the afternoon", "The lamps on the bridge are lit at 6:45 in the evening".
- A minutes card per pair for support: a ring of 5, 10, 15 ... 60 to sit around the card clock, one number beside each clock number.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Point to each number on the teaching clock and count in fives together: 1 is 5 minutes, 2 is 10 minutes, and so on round to 12, which is 60 minutes or "o'clock". "The minute hand goes all the way round in one hour. Each number is 5 more minutes."

✎ Main activity (30 min)

1. **O'clock and half past (8 min).** "The short hand is the hour hand. The long hand is the minute hand." Set 3:00: the minute hand on 12, the hour hand on 3. Move the minute hand slowly round to 6 and show the hour hand moving too. "Half an hour has gone. It is half past 3, or 3:30. The hour hand is halfway between 3 and 4." Show the digital cards 3:00 and 3:30.
2. **Five-minute jumps (12 min).** Set 4:25 on the teaching clock. "The minute hand points to 5. Count in fives: 5, 10, 15, 20, 25. The hour hand has gone past 4 but is not yet halfway to 5. It is 4:25." Pairs set each time on their card clock and match the digital card: 7:40 (minute hand on 8, hour hand more than halfway from 7 to 8), 10:05 (minute hand on 1, hour hand just past 10), 11:55 (minute hand on 11, hour hand

almost at 12 but still 11 o'clock and some minutes). Partners take turns: one sets a time, the other reads it and writes it.

3. **a.m. and p.m. (10 min).** "There are two of every time each day: one in the morning and one in the afternoon or evening. Times from midnight until midday are a.m. Times from midday until midnight are p.m." Hand out the Little Hill day cards. Pairs set each time on the card clock, write it with a.m. or p.m., then the class puts the cards in order through the day:
    - 7:15 a.m. Hen opens the bakery;
    - 8:40 a.m. the bus leaves for school;
    - 12:10 p.m. lunch is ready at the market;
    - 3:25 p.m. Mayor Hen rings the town bell;
    - 6:45 p.m. the lamps on the bridge are lit.

✓ Discussion questions (with answers)

- How do you know which hand is which? The hour hand is shorter; the minute hand is longer.
- At 11:55, the hour hand is almost on 12. Why do we say 11:55 and not 12:55? The hour hand has not reached 12 yet, so it is still 11 and some minutes.
- Is lunch at 12:10 a.m. or p.m.? p.m., because it is just after midday.

✓ Closing check (10 min, with answers)

Set 2:35 on the teaching clock. "What time is it? Write it." (2:35.) "Show me 6:10 on your clock." (Minute hand on 2, hour hand just past 6.) "Goat reads a story before bed at 7:30. Is that a.m. or p.m.?" (p.m.) Note who reads the hour from the number the hour hand has just passed, and who reads the nearest number.

Vocabulary

clock, hour hand, minute hand, o'clock, half past, minutes, a.m., p.m., morning, afternoon, evening, midday, midnight

Differentiation

- **Support:** set only o'clock, half past, and times where the minute hand points to 1, 2 or 3 (5, 10 or 15 minutes past). Put the minutes card around the card clock.
- **Stretch:** "The bus leaves at 8:40. Hen arrives 10 minutes later. What time does Hen arrive?" (8:50.) Talk about "quarter past" and "quarter to" as other ways to say 15 and 45 minutes.
- Children who are blind or have low vision feel the raised numbers and the hands of the teaching clock or their card clock.

⚠ Safety

An adult fixes the paper fasteners and tapes over the sharp ends before the lesson. Never ask children about their own wake, meal or bed times; the times belong to the Little Hill characters.

Teacher notes and misconceptions

- The most common mistake is reading the hour from the nearest number when the minute hand is past 30, so 9:50 is read as 10:50. Ask "Has the hour hand reached 10 yet?"
- Some children mix up the hands. Giving the minute hand a different colour and a different feel (a strip of tape along it) and saying "long hand, minutes" helps.
- Times exactly at 12:00 can confuse even adults (12:00 midday is often written 12:00 p.m.), so the lesson avoids them.
- Many children see more digital clocks than clocks with hands. Keep a clock with hands in the classroom and read it together each day.

Lesson 4: Counting coins at the Little Hill market

Unit overview - Previous: Lesson 3 - Next: Lesson 5

Back to the grade 2 year plan

About 45 minutes.

Goal

Children name pennies, nickels, dimes and quarters and their values, find the value of a set of coins by counting on from the biggest, write amounts with the ¢ sign, and solve one-step story problems with made-up prices.

Materials

- Card coins per pair, too large to swallow, printed on both sides with the value raised (glued string or thick glue) so it can be felt: 4 quarters, 6 dimes, 6 nickels and 10 pennies, plus one card dollar bill. Draw the coins in the same order of size as real coins (the dime smallest, then the penny, the nickel and the quarter).
- Little Hill market price cards with made-up prices (an example, not real prices): pear 35¢, ribbon 20¢, pencil 15¢, packet of seeds 50¢, toy kite 75¢.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Count together in fives to 50 and in tens to 100, as in term 2. "Today we will use counting in fives and tens to count money."

✎ Main activity (30 min)

1. **Meeting the coins (8 min).** Show each card coin and say its value: a penny is 1 cent (1¢), a nickel is 5¢, a dime is 10¢ and a quarter is 25¢. A dollar is 100¢, written $1. Pairs sort their coins into four piles. "Which coin is the smallest?" (The dime.) "Which coin is worth the most?" (The quarter.) "Is the smallest coin worth the least?" (No: a dime is worth more than a penny or a nickel.)
2. **Counting a handful (12 min).** Model counting on from the biggest value: quarter, dime, dime, penny: "25, 35, 45, 46. That is 46¢." Pairs take turns to give their partner a set of coins to count, then try these together:
    - 2 dimes and 3 pennies (23¢);
    - 3 dimes, 2 nickels and 4 pennies (dimes 10, 20, 30; nickels 35, 40; pennies 41, 42, 43, 44: 44¢);
    - a quarter, a dime and a nickel (40¢);
    - 4 quarters (25, 50, 75, 100: 100¢, which is $1).

3. **The Little Hill market (10 min).** Put out the price cards. Pairs pay for each thing with card coins in one way, then find a second way (the pear, 35¢: a quarter and a dime, or 3 dimes and a nickel). Then story problems, solved with coins and written as a number sentence:
    - "Duck has 2 quarters and buys a pear. How much money is left?" (50¢ - 35¢ = 15¢.)
    - "Goat buys a packet of seeds and a pencil. How much is that?" (50¢ + 15¢ = 65¢.)
    - "Mouse has 60¢. The toy kite costs 75¢. How much more does Mouse need?" (75¢ - 60¢ = 15¢.)

✓ Discussion questions (with answers)

- Why do we start counting with the coin worth the most? It is quicker: big jumps first, then small ones.
- How many dimes make a dollar? 10, because 10 tens make 100.
- Can two children have different coins but the same amount of money? Yes: a quarter and a nickel is 30¢, and so are 3 dimes.

✓ Closing check (10 min, with answers)

Show 2 quarters, a dime and a penny. "How much money is this?" (61¢.) "Show me 30¢ in two different ways." (For example 3 dimes; a quarter and a nickel; 6 nickels.) "Hen has 50¢ and buys a packet of seeds. How much is left?" (0¢.) Note who counts on from the biggest coin and who counts every coin as 1.

Vocabulary

penny, nickel, dime, quarter, dollar, cent, ¢, $, value, worth, price, cost, left, more

Differentiation

- **Support:** use only dimes and pennies first, which works like tens and ones (3 dimes and 4 pennies is 34¢), then add nickels.
- **Stretch:** "Find all the ways to pay 25¢ with dimes, nickels and pennies if you use no more than 5 coins." (2 dimes and 1 nickel; 1 dime and 3 nickels; 5 nickels.)
- Children who are blind or have low vision feel the raised values on the card coins. Sorting by size alone does not work because the dime is the smallest coin.

⚠ Safety

Use card coins, not real coins: real coins are a choking risk and are handled by many people. Never ask children about family money, pocket money or what things cost at home; the prices belong to the Little Hill market.

Teacher notes and misconceptions

- Many children think a bigger coin is worth more, so the nickel seems worth more than the dime. Keep asking "What is it worth?" rather than "Which is bigger?"
- Some children count every coin as one. Ask "How many cents is this coin?" before counting.
- These are United States coins. Schools in other countries should use their own coins and change the values and prices.
- Writing amounts with a decimal point ($0.42) is not taught here; amounts under a dollar are written with ¢.

Lesson 5: Sides, corners and a shape bar graph

Unit overview - Previous: Lesson 4

Back to the grade 2 year plan

About 45 minutes.

Goal

Children name triangles, quadrilaterals, pentagons and hexagons by counting sides and corners, say how many faces a cube has, put shapes together to make new shapes, and make and read a bar graph with four categories.

Materials

- Card shapes per pair in different sizes and turned different ways: triangles, quadrilaterals (including squares, rectangles and some with no equal sides), pentagons and hexagons.
- Pattern blocks too large to swallow, per pair: 6 triangles, 2 trapezoids, 2 rhombuses (diamonds) and 1 hexagon.
- A cube per pair, such as a large wooden or foam block.
- A shape bag per pair holding exactly 19 card shapes: 5 triangles, 7 quadrilaterals, 3 pentagons and 4 hexagons.
- Squared paper (2 cm squares) and dot paper per child; card squares and glue for raised bar graphs. Small stickers for marking corners.
- A bar graph for the board, also made with raised card squares: boats on the Little Hill pond, with sailing boats 3, rowing boats 5 and canoes 2.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

"In grade 1 we found out what makes a triangle a triangle." Hold up a long, thin triangle and a square. "How did we know?" (Count the straight sides and the corners: 3 for a triangle.) "Today we count sides and corners to name more shapes."

✎ Main activity (30 min)

1. **Count and name (10 min).** Pairs take turns to pick a card shape, run a finger along each side while counting, then touch each corner. Name each group as it is found: 3 sides and 3 corners is a triangle; 4 sides and 4 corners is a quadrilateral (squares and rectangles are quadrilaterals too); 5 is a pentagon; 6 is a hexagon. "Does turning the shape or making it bigger change its name?" (No.) Then hold up the cube: pairs count its flat faces (6, all squares the same size) and its corners (8). Each child draws a shape with 5 corners on dot paper and a partner checks it is a pentagon.

2. **Making new shapes (8 min).** With pattern blocks: "Put 2 triangles together to make a shape with 4 sides." (A rhombus, which is a quadrilateral.) "Can you cover a trapezoid with triangles?" (Yes, 3.) "Can you cover the hexagon with triangles?" (Yes, 6.) "With trapezoids?" (Yes, 2.) Pairs lay the pieces on top of the hexagon to check, then count the sides of each new shape they make.
3. **A shape bar graph (12 min).** Each pair tips out its shape bag and sorts the shapes into four rows by name, one card for each shape, lined up at the left like a picture graph. Then they make a bar graph on squared paper: one square for each shape, filled in or covered with a card square, with the four names written along the bottom. Ask:
    - "How many shapes altogether?" (5 + 7 + 3 + 4 = 19.)
    - "Which shape is there least of?" (Pentagons.)
    - "How many more quadrilaterals than pentagons?" (7 - 3 = 4.)
    - "How many triangles and hexagons together?" (5 + 4 = 9.)

✓ Discussion questions (with answers)

- Is a square a quadrilateral? Yes: it has 4 sides and 4 corners.
- A shape has 6 sides but they are not all the same length. Is it a hexagon? Yes: it has 6 sides and 6 corners.
- Why do the bars start at the same line? So we can compare them fairly, just like lining up strips at one end before measuring.

✓ Closing check (10 min, with answers)

Hold up a pentagon with sides of different lengths. "What is this shape?" (A pentagon: 5 sides and 5 corners.) "Draw a shape with 4 corners. What is it called?" (A quadrilateral.) Show the boats bar graph. "How many boats altogether?" (10.) "How many more rowing boats than canoes?" (3.) Note who counts sides carefully and who names shapes by how they look.

Vocabulary

side, corner, triangle, quadrilateral, pentagon, hexagon, cube, face, rhombus, trapezoid, bar graph, picture graph, altogether, how many more

Differentiation

- **Support:** sort triangles and quadrilaterals first, then add the other shapes. Put a small sticker on each corner as it is counted.
- **Stretch:** count the edges of the cube (12). "How many triangles would cover 2 hexagons?" (12.)
- Children who are blind or have low vision feel the sides and corners of the card shapes and pattern blocks, and read the bar graph by feeling the raised card squares in each bar.

⚠ Safety

Use pattern blocks and cubes too large to swallow. Cubes stay on tables.

Teacher notes and misconceptions

- Many children think a shape turned on its corner is a different shape (a square "becomes a diamond"). Turn shapes and ask "Did the number of sides change?"
- Some children only call a shape a hexagon if all its sides are the same. Show irregular hexagons often.
- Counting corners on a shape with many sides is easy to get wrong; touching each corner, or putting a small sticker on it, helps.
- Line plots, splitting rectangles into rows of squares, and halves, thirds and fourths are not taught in this unit (see Known gaps in the unit overview).

Grade 3 Mathematics: Equal Groups and Arrays (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 11 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard. Check activities against your school's rules.
>
> Text-checked by three other agents (Gemini 3.8 Flash, Gemini 4 Argon and Gemini 4 Carbon); see Known gaps.

Back to the grade 3 year plan - Term 1. Builds on grade 2: Adding and taking away within 100 and Hundreds, tens and ones to 1000

Big idea

When things come in equal groups, we can find the total without counting every one: 5 groups of 7 is 5 × 7. Putting the groups in rows makes an array, and turning an array round shows that 5 × 7 and 7 × 5 give the same total. Dividing works the other way: sharing 24 into 4 equal groups, or making groups of 4 from 24, and both can be found by asking "4 times what is 24?"

Outcomes

By the end of the unit most children can:

1. Read, write, compare and order numbers to 1000, and add or take away within 1000 with a method they can explain using hundreds, tens and ones.
2. Describe a collection of equal groups as "5 groups of 7" and write it as 5 × 7, and tell a short story that fits a multiplication like 5 × 7.
3. Build and draw arrays, write a multiplication for an array, and explain why turning an array round does not change the total (4 × 6 = 6 × 4).
4. Solve a division in both ways, sharing (24 shared into 4 equal groups is 6 in each group) and grouping (24 put into groups of 4 makes 6 groups), and write it as 24 ÷ 4 = 6.
5. Solve one-step story problems with equal groups and arrays within 100, and find the missing number in sentences like 8 × □ = 48 or □ = 18 ÷ 3.

Prior knowledge

The grade 2 units: hundreds, tens and ones to 1000; adding and taking away within 100 using tens and ones; skip-counting in 2s, 5s, 10s and 100s; odd and even numbers; and finding the total of a small array by adding equal rows (for example 4 + 4 + 4). Children still building these can use numbers to 30 and groups of 2, 5 and 10 in every lesson.

Lessons

Each lesson is about 45 minutes: a short whole-class start, paired work with counters or cubes, and a closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so

each idea should come back in short daily practice (5 minutes of skip-counting, one array or one story problem).

1. Hundreds, tens and ones: a check of grade 2
2. Equal groups and the times sign
3. Arrays, and turning them round
4. Dividing: sharing and grouping
5. Story problems and missing numbers

Materials (whole class unless noted)

- Counters or linking cubes too large to swallow, about 60 per pair (every lesson), in two colours or two kinds of object so colour is not the only clue.
- Place-value blocks or bundles per pair: 6 hundreds (flats or ten towers of ten), 15 tens and 20 ones (lesson 1). Straw bundles wrapped with tape (no rubber bands, which may contain latex) work as well.
- A hundreds-tens-and-ones mat per pair, with raised lines (glued string) between the columns (lesson 1).
- Six paper plates or small trays per pair, for groups (lessons 2 and 4).
- Squared paper, 1 cm or larger, and a large-print or raised-line version (lessons 3 and 5); a sheet of card per pair for covering part of an array (lesson 3).
- A 10 by 10 pegboard or egg boxes per pair as an optional tactile array (lesson 3).
- Two sets of digit cards 0 to 9 per pair, so children can build and point to answers (every lesson).
- Story-problem cards for lesson 5 (see lesson 5), with a small picture on each card.
- Whiteboards and pens, or paper and pencils, for drawings and number sentences.

⚠ Safety and access

- Use counters and cubes too large to swallow; no rubber bands.
- Plates or trays, cube towers and raised lines let children who are blind or have low vision feel each group and each row; pegboards or egg boxes make arrays tactile.
- Pointing to digit cards, moving counters, sign, writing or a communication device all count as answers. Story problems are read aloud, so reading is not what is being checked.
- No public timing, races or times-table tests. Knowing products from memory is an end-of-year aim; this unit is about what multiplying and dividing mean.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week:

- "What is 345 + 278? How did you work it out?" (623: 3 hundreds and 2 hundreds are 5 hundreds; 4 tens and 7 tens are 11 tens; 5 ones and 8 ones are 13 ones, which is 1 ten and 3 ones; so 6 hundreds, 2 tens and 3 ones.)
- Show 4 plates with 3 counters on each. "How many groups? How many in each group? Write it as a times sentence." (4 groups of 3; 4 × 3 = 12.)
- Show an array of 3 rows of 6. "Write a times sentence for it. What if I turn it round?" (3 × 6 = 18; turned round it is 6 rows of 3, 6 × 3 = 18, the same counters.)

- "Share 20 counters equally onto 5 plates. How many on each?" (4; 20 ÷ 5 = 4.) "Now put 20 counters into groups of 5. How many groups?" (4; 20 ÷ 5 = 4.)
- "Hen puts 6 eggs in each box. Hen fills 4 boxes. How many eggs?" (24; 4 × 6 = 24.) "What number goes in the box: 7 × □ = 35?" (5.)

Record yes, with help or not yet for each part. A child who gets the right total by counting in ones has the answer but not yet the equal-groups idea; note that rather than marking it wrong.

Sources

- Common Core State Standards for Mathematics, Grade 3 (read to check what is usually expected in grade 3, not to claim alignment), CCSSO PDF: https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf
  - Page 21, grade 3 introduction: the first critical area is "developing understanding of multiplication and division and strategies for multiplication and division within 100".
  - Page 23, 3.OA.A.1: "Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each."
  - 3.OA.A.2: interpret 56 ÷ 8 "as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each."
  - 3.OA.A.3: "Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities".
  - 3.OA.A.4: "Determine the unknown whole number in a multiplication or division equation relating three whole numbers."
  - 3.OA.B.5: "Apply properties of operations as strategies to multiply and divide", with the footnote "Students need not use formal terms for these properties."
  - 3.OA.B.6: "Understand division as an unknown-factor problem."
  - Page 24, 3.NBT.A.2: "Fluently add and subtract within 1000 using strategies and algorithms based on place value".

Known gaps and cautions

- 3.OA.C.7 (by the end of grade 3, know from memory all products of two one-digit numbers) is a whole-year aim; this unit starts it with skip-counting in 2s, 3s, 4s, 5s and 10s, and the year plan puts more times-table facts in term 2.
- Measurement quantities in story problems (3.OA.A.3), two-step problems (3.OA.D.8), the grouping property (associative) and splitting a times fact (distributive) are left for later terms. Rounding to the nearest 10 or 100 (3.NBT.A.1) is not taught here.
- Lesson 1 is a check of grade 2 work, not new teaching; if most children are secure, it can be shortened to one short session.
- The word "commutative" is the teacher's; children only need the idea and may say "turning it round". Children do learn the word "array".
- The names in story problems (Hen, Goat, Bear, Duck, Mouse, Crab) are the made-up Little Hill characters used in other units. Swap in names and objects that suit your class, but avoid real children's belongings or families.
- The lesson order, timings and examples are my choice, not taken from a source.
- Three other agents (Gemini 3.8 Flash at 4f59beb, reporting no errors; Gemini 4 Argon at 4f59beb and 4720d19, and Gemini 4 Carbon at 4720d19, each reporting the same two material-count errors) text-checked this unit on 2026-11-11. The errors (too few place-value blocks for 345 + 278, and 50 counters for 6 groups of 10) were fixed in ea1b95b, and Gemini 4 Argon and Gemini 4 Carbon re-checked that fix the same day and reported no

remaining errors. Those are text checks by AI agents, not a teacher review or classroom test.

Lesson 1: Hundreds, tens and ones: a check of grade 2

Unit overview - Next: Lesson 2

About 45 minutes.

Goal

Children show three-digit numbers with hundreds, tens and ones, compare them, and add or take away within 1000 with a method they can explain. This lesson checks the grade 2 work before multiplication starts; the teacher notes who needs more practice.

Materials

- Place-value blocks or bundles per pair: 6 hundreds, 15 tens and 20 ones.
- A hundreds-tens-and-ones mat per pair, with raised lines between the columns.
- Two sets of digit cards 0 to 9 per pair.
- Counters or linking cubes, for children who prefer to use them for ones.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Skip-counting. The class counts together in 10s from 40 to 150, then in 100s from 100 to 1000, then in 5s from 0 to 60. Children who prefer can tap the table or hold up digit cards instead of speaking.

✎ Main activity (32 min)

1. **Build and compare (10 min).** The teacher says a number; pairs build it on the mat with blocks, then make it with digit cards. Use 345, 278, 406 and 460. Ask: "Which is bigger, 406 or 460?" (460: both have 4 hundreds, and 6 tens is more than 0 tens.) "Put 345, 278 and 406 in order, smallest first." (278, 345, 406.)
2. **Add with blocks (12 min).** Pairs build 345 and 278 and push them together. Ones: 5 + 8 = 13, so swap 10 ones for 1 ten, leaving 3 ones. Tens: 4 + 7 + 1 = 12 tens, so swap 10 tens for 1 hundred, leaving 2 tens. Hundreds: 3 + 2 + 1 = 6. Total 623. Then pairs do 156 + 238 (394; one swap of ones) on their own and write how they did it in numbers or a drawing.
3. **Take away with blocks (10 min).** Pairs build 532 and take away 217. There are only 2 ones, so swap 1 ten for 10 ones (12 ones); 12 - 7 = 5 ones; 2 - 1 = 1 ten; 5 - 2 = 3 hundreds. Answer 315. Check by adding: 315 + 217 = 532. Then 400 - 150 (250) for pairs who are ready.

✓ Discussion questions (with answers)

- Why do we swap 10 ones for 1 ten? Ten ones and one ten are the same amount; the swap keeps the number the same but makes it easier to read.
- How can adding check a take-away? If 532 - 217 = 315, then 315 + 217 must make 532 again.
- Is 406 the same as 460? No: 406 has no tens and 6 ones; 460 has 6 tens and no ones.

✓ Closing check (8 min, with answers)

Each child answers on a whiteboard, with digit cards or by pointing on the mat: "Show 507 with blocks" (5 hundreds, 0 tens, 7 ones); "Which is bigger, 389 or 398?" (398); "345 + 278" (623). One child at a time quietly tells the teacher how they did the adding.

Vocabulary

hundreds, tens, ones, digit, swap (exchange), compare, add, take away

Differentiation

- **Support:** work within 100 with tens and ones only, then add hundreds.
- **Stretch:** find two different three-digit numbers that add to 623 with one swap, and explain how you know.
- The mat's raised lines let children feel the columns. Blocks can be counted by touch; digit cards can be read aloud by a partner or made in large print or braille.

⚠ Safety

Use blocks and counters too large to swallow. Tape bundles rather than using rubber bands, which may contain latex.

Teacher notes and misconceptions

- Some children read 406 as "forty-six". Building it with blocks and leaving the tens column empty helps them see the zero.
- Taking the smaller digit from the larger in each column (writing 532 - 217 as 325) is a common mistake. Checking by adding shows it is wrong.

Lesson 2: Equal groups and the times sign

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 45 minutes.

Goal

Children make equal groups, describe them as "4 groups of 3", write them as 4 × 3, find the total by skip-counting or repeated adding, and tell a short story that fits a multiplication.

Materials

- Six paper plates or small trays per pair.
- Counters or linking cubes, about 60 per pair, in two colours or two kinds of object.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Skip-counting in 2s to 20, in 5s to 50 and in 10s to 100. Children can say the numbers, tap them, or hold up digit cards.

✎ Main activity (32 min)

1. **Groups of (10 min).** The teacher puts 3 counters on each of 4 plates. "How many plates? How many on each plate?" Children say "4 groups of 3". Count in 3s together, touching each plate: 3, 6, 9, 12. Write 3 + 3 + 3 + 3 = 12, then show the short way: 4 × 3 = 12, read as "4 groups of 3" or "4 times 3". Then put 3 counters on one plate and 5 on another and ask: "Is this equal groups?" (No; the groups are not the same size.)

2. **Make it, write it (12 min).** Pairs build each of these with plates and counters, find the total and write the multiplication:

   - 5 groups of 2 (5 × 2 = 10).
   - 3 groups of 5 (3 × 5 = 15).
   - 6 groups of 10 (6 × 10 = 60).
   - 4 groups of 4 (4 × 4 = 16).

   Then the teacher writes 2 × 6 and pairs build it (2 plates of 6 = 12).

3. **Story time (10 min).** The teacher tells an example story, not from a source: "Duck has 3 baskets. Each basket has 5 apples. How many apples?" (3 × 5 = 15.) Pairs pick a multiplication from step 2 and make up their own short story about Little Hill characters, telling it aloud, drawing it or acting it with counters. The partner builds the story with plates to check it fits.

✓ Discussion questions (with answers)

- What makes groups "equal"? Every group has the same number of things.
- In 4 × 3, what does the 4 tell us and what does the 3 tell us? In this lesson, 4 is the number of groups and 3 is the number in each group.
- Why is 6 × 10 quick to work out? Counting in 10s six times is 10, 20, 30, 40, 50, 60.

✓ Closing check (8 min, with answers)

The teacher puts 3 counters on each of 4 plates in front of each child (or pair) in turn. The child says or shows how many groups (4), how many in each (3), the total (12) and the multiplication (4 × 3 = 12), with digit cards or on a whiteboard. Then: "Write 5 groups of 2 with the times sign" (5 × 2 = 10).

Vocabulary

equal groups, groups of, times, multiply, total, skip-count

Differentiation

- **Support:** use groups of 2, 5 and 10 only, and count every counter if needed before skip-counting.
- **Stretch:** find all the ways to put 12 counters into equal groups (1 × 12, 2 × 6, 3 × 4, 4 × 3, 6 × 2, 12 × 1).
- Plates with raised rims let children feel each group. Stories can be told, signed, drawn or acted; the child's own way counts.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Some children add the two numbers (4 × 3 = 7). Going back to plates and counters, and saying "4 groups of 3", usually fixes this.
- Different books read 4 × 3 as "4 groups of 3" or "3, 4 times". This unit uses "groups of" first; lesson 3 shows that both orders give the same total.

Lesson 3: Arrays, and turning them round

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 45 minutes.

Goal

Children build and draw arrays, write a multiplication for an array, and explain why turning an array round gives the same total, so 4 × 6 = 6 × 4.

Materials

- Counters or linking cubes, about 60 per pair, in two colours or two kinds of object.
- Squared paper, 1 cm or larger, and a large-print or raised-line version.
- A sheet of card per pair, for covering part of an array.
- Optional: a 10 by 10 pegboard or egg boxes per pair, as a tactile array.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Skip-counting in 3s to 30 and in 4s to 40, together. Children can say, tap or show the numbers with digit cards.

✎ Main activity (32 min)

1. **Rows of (10 min).** The teacher builds 3 rows of 6 counters. "How many rows? How many in each row?" This is an array: equal groups set out in rows. Count in 6s along the rows: 6, 12, 18, so 3 × 6 = 18. Now turn the whole array a quarter turn. "How many rows now? How many in each?" (6 rows of 3.) Count in 3s: 3, 6, 9, 12, 15, 18. Same counters, same total: 6 × 3 = 18.

2. **Build, turn and draw (12 min).** Pairs build each array, write both multiplications, then colour it on squared paper:

    - 2 rows of 5 (2 × 5 = 10 and 5 × 2 = 10).
    - 4 rows of 6 (4 × 6 = 24 and 6 × 4 = 24).
    - 3 rows of 7 (3 × 7 = 21 and 7 × 3 = 21).

    Children can use the pegboard or egg boxes instead of counters.

3. **What's under the card? (10 min).** One child builds an array and hides it under the card, then slides the card to show just one row and says how many rows there are. The partner works out the total (for example, one row of 5 showing and 4 rows altogether: 4 × 5 = 20), then lifts the card to check by counting. Swap roles.

✓ Discussion questions (with answers)

- Why does turning an array round not change the total? No counters are added or taken away; they are only looked at another way.
- If you know 3 × 7 = 21, what else do you know? 7 × 3 = 21.
- Is 5 × 2 the same as 5 + 2? No: 5 × 2 is 5 groups of 2, which is 10.

✓ Closing check (8 min, with answers)

Each child builds 3 rows of 6 (or the teacher builds it in front of them). The child says or shows the multiplication (3 × 6 = 18), then turns it round and gives the other one (6 × 3 = 18), with digit cards, a whiteboard or by pointing. Then: "2 × 9 = 18. What is 9 × 2?" (18).

Vocabulary

array, row, column, turn round, multiply, same total

Differentiation

- **Support:** use arrays with rows of 2, 5 or 10 and count along each row.
- **Stretch:** find every array that uses exactly 24 counters (1 × 24, 2 × 12, 3 × 8, 4 × 6 and each one turned round).
- The pegboard, egg boxes and raised-line squared paper let children feel rows and columns. Counters come in two colours or two kinds, so colour is not the only clue when rows are marked.

⚠ Safety

Use counters and pegs too large to swallow.

Teacher notes and misconceptions

- The idea that order does not change the answer is called the commutative property. This is the teacher's word only; the standards say children need not use formal terms for these properties.
- Some children count every counter each time. Ask them to count one row, then skip-count by rows.

Lesson 4: Dividing: sharing and grouping

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 45 minutes.

Goal

Children solve a division in two ways, sharing (how many in each group?) and grouping (how many groups?), write it with the division sign, and link it to a multiplication they know.

Materials

- Six paper plates or small trays per pair.
- Counters or linking cubes, about 60 per pair, in two colours or two kinds of object.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

Quick multiplication from lesson 3: the teacher says "4 rows of 6"; children show the total (24) with digit cards, fingers or a whiteboard. Repeat with 3 rows of 5 (15) and 2 rows of 10 (20).

✎ Main activity (32 min)

1. **Sharing (10 min).** The teacher puts out 4 plates and 24 counters. "Share these fairly onto 4 plates." Deal one at a time onto each plate until none are left. Each plate has 6. Write 24 ÷ 4 = 6, read as "24 shared into 4 groups is 6 in each". Pairs then share 15 onto 3 plates (5 each) and 20 onto 5 plates (4 each).
2. **Grouping (10 min).** Start again with 24 counters, but now the question is different: "Put them in groups of 4. How many groups?" Make groups of 4 on plates until none are left: 6 groups. Write 24 ÷ 4 = 6 again, read as "24 in groups of 4 makes 6 groups". Pairs then make groups of 5 from 20 (4 groups) and groups of 2 from 12 (6 groups).
3. **Times or divide? (12 min).** For each division pairs find the answer both ways if they can, and write the multiplication that checks it:

    - 20 ÷ 5 = 4, because 5 × 4 = 20.
    - 18 ÷ 3 = 6, because 3 × 6 = 18.
    - 30 ÷ 10 = 3, because 10 × 3 = 30.
    - 16 ÷ 4 = 4, because 4 × 4 = 16.

    Then ask: "24 ÷ 4: what number times 4 makes 24?" (6.)

✓ Discussion questions (with answers)

- What is different about sharing and grouping? In sharing we know how many groups and find how many in each; in grouping we know how many in each and find how many groups.
- Why do both give the same answer for 24 ÷ 4? Both are about the same array: 4 groups of 6 and 6 groups of 4 both make 24.
- How can multiplying check a division? If 18 ÷ 3 = 6, then 3 × 6 must make 18.

✓ Closing check (8 min, with answers)

Each child (or pair, with the teacher watching) is given 20 counters and 5 plates. "Share 20 onto 5 plates. How many on each?" (4.) Then "Put 20 in groups of 5. How many groups?" (4 groups.) The child shows 20 ÷ 5 = 4 with digit cards or on a whiteboard.

Vocabulary

share, fairly, equal groups, divide, division sign, how many in each, how many groups

Differentiation

- **Support:** divide numbers to 20 by 2, 5 and 10, dealing one counter at a time.
- **Stretch:** share 13 counters onto 4 plates and explain what happens (3 each, with 1 left over; leftovers come in a later grade).
- Plates with raised rims let children feel each group. Children can deal counters by touch and count each plate aloud or by signing.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- The same sign, ÷, is used for sharing and grouping. Children do not need to name which one they used, but should be able to show either with counters.
- Some children think the bigger number must go second (4 ÷ 24). Reading it aloud as "24 shared into 4" helps.

Lesson 5: Story problems and missing numbers

Unit overview - Previous: Lesson 4

About 45 minutes.

Goal

Children solve one-step story problems with equal groups and arrays within 100, choosing to multiply or divide, and find the missing number in sentences like 8 × □ = 48 or □ = 18 ÷ 3.

Materials

- Story-problem cards per pair, each with a small picture and the answer written small on the back. Six cards (examples written for this unit, not from a source; the characters live in the made-up town of Little Hill):
    1. Hen has 4 boxes. Each box holds 6 eggs. How many eggs?
    2. Goat plants carrots in 3 rows with 8 in each row. How many carrots?
    3. Bear shares 30 berries onto 5 plates. How many berries on each plate?
    4. Duck puts 28 crackers into bags of 4. How many bags?
    5. Mouse sets out 5 rows of chairs with 9 in each row. How many chairs?
    6. Crab has 40 shells and puts 10 in each bucket. How many buckets?
- Counters or linking cubes, about 60 per pair, in two colours or two kinds of object.
- Squared paper, 1 cm or larger, and a large-print or raised-line version.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.

Steps

✎ Opening (5 min)

The teacher says a fact; children show the answer with digit cards, fingers or a whiteboard: 5 × 3 (15), 2 × 8 (16), 20 ÷ 4 (5), 10 × 7 (70).

✎ Main activity (32 min)

1. **Solve together (8 min).** Read card 1 aloud. "Is it equal groups? How many groups? How many in each?" (4 groups of 6.) Children solve it with counters, a drawing or skip-counting: 4 × 6 = 24 eggs. Then read card 3: "Do we know how many groups, or how many in each?" (5 groups; find how many in each), so 30 ÷ 5 = 6 berries.
2. **Story cards (14 min).** Pairs solve the other cards in any order, using counters, squared paper or a drawing, and write the number sentence. Answers: card 2, 3 × 8 = 24 carrots; card 4, 28 ÷ 4 = 7 bags; card 5, 5 × 9 = 45 chairs; card 6, 40 ÷ 10 = 4 buckets. The partner checks the back. A partner can read the cards aloud.

3. **Missing numbers (10 min).** The teacher writes a sentence with a box. Pairs find the missing number with counters or arrays and show it with digit cards:
    - 8 × □ = 48 (6).
    - 4 × □ = 20 (5).
    - □ × 3 = 27 (9).
    - □ = 18 ÷ 3 (6).
    - 35 ÷ 7 = □ (5).

    Ask: "How can a times fact help with 8 × □ = 48?" (Count in 8s until you reach 48: 8, 16, 24, 32, 40, 48 is six 8s.)

✓ Discussion questions (with answers)

- How do you know whether to multiply or divide? If we know how many groups and how many in each, we multiply to find the total; if we know the total and one of the others, we divide.
- What does the box mean in 8 × □ = 48? It stands for a number we do not know yet; here it is 6.
- Is card 2 about groups or an array? An array: rows of carrots. Rows work just like groups.

✓ Closing check (8 min, with answers)

One child at a time, with the teacher. The teacher reads aloud: "Hen has 4 boxes with 6 eggs in each. How many eggs?" (24.) Then shows 7 × □ = 35 and asks for the missing number (5). The child answers by saying, signing, pointing to digit cards, writing or using counters, and shows or tells how they got it.

Vocabulary

story problem, number sentence, multiply, divide, missing number, unknown

Differentiation

- **Support:** use cards 1, 3 and 6 with counters or a drawing; missing numbers with 2, 5 and 10 only.
- **Stretch:** write a story card of your own for 6 × 7 and one for 42 ÷ 6 (both use 42), and swap with another pair.
- Cards can be read aloud by a partner or made in large print or braille. Raised-line squared paper lets children feel rows of an array.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Children often look for a key word ("each", "share") instead of thinking about the story. Ask them to act it out or draw it first.
- Children may write 4 ÷ 28 for card 4. Reading it aloud as "28 in bags of 4" helps.
- Fluency with all times facts within 100 is a whole-year aim; keep up short daily practice after this unit.

Grade 3 Mathematics: Fractions (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 12 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard. Check activities against your school's rules.
>
> Text-checked by two other agents (Gemini 3.8 Flash and Gemini 4 Argon); see Known gaps.

Back to the grade 3 year plan - Term 2. Builds on grade 2 Measuring, time, money and shapes (halves, thirds and fourths of shapes) and grade 3 Equal groups and arrays

Big idea

A fraction is a number made from equal parts of one whole. Split a strip into 4 equal parts and each part is one quarter, written 1/4; three of those parts make 3/4. Fractions sit on the number line between whole numbers, two fractions can name the same amount (1/2 = 2/4), and we can only compare fractions fairly when they are parts of the same whole.

Outcomes

By the end of the unit most children can:

1. Explain that a fraction needs equal parts of one whole, and name one half, one third, one quarter, one sixth and one eighth (1/2, 1/3, 1/4, 1/6, 1/8) of a strip or a shape.
2. Build, read and write fractions such as 3/4 as "3 parts of size 1/4", and know that 4/4 is one whole and that 3 wholes can be written 3/1.
3. Place fractions on a number line from 0 to 1, and from 0 to 2 in quarters, by marking off equal lengths from 0.
4. Compare two fractions with the same top number or the same bottom number, write the result with >, < or =, explain it with strips, and say why the comparison only works when both fractions are parts of the same whole.
5. Find simple equal fractions, such as 1/2 = 2/4 and 2/3 = 4/6, and show why with strips or a number line.
6. Give more times-table facts within 100 (2s, 3s, 4s, 5s, 6s, 8s and 10s), from memory or with a quick strategy such as doubling.

Prior knowledge

The grade 2 shapes lesson: splitting circles and rectangles into two, three or four equal shares and calling them halves, thirds and fourths. The grade 3 term 1 unit: equal groups, arrays and skip-counting in 2s, 3s, 4s, 5s and 10s. Children who are not yet secure can work with halves and quarters only in every lesson.

Lessons

Each lesson is about 45 minutes: a 5-minute times-table start, paired work with paper strips or fraction strips, and a closing check. Split a lesson over two days if needed; the unit runs over

about 4 weeks, so each idea should come back in short daily practice (one times-table fact, one fraction to build or one comparison).

1. Equal parts and unit fractions
2. Counting in unit fractions
3. Fractions on a number line
4. Comparing fractions
5. Equal fractions and fraction stories

Materials (whole class unless noted)

- Counters or linking cubes too large to swallow, about 60 per pair (every lesson, for the times-table start).
- Two sets of digit cards 0 to 9 per pair (every lesson).
- Whiteboards and pens, or paper and pencils (every lesson).
- Plain paper strips 24 cm long and about 4 cm wide, 5 per child, two of them pre-marked with pencil lines in thirds (every 8 cm) and sixths (every 4 cm) (lesson 1).
- Paper squares about 15 cm across, 3 per child (lesson 1) and 1 more per child (lesson 5).
- Three check strips made by the teacher, each 24 cm long, with the lines drawn in thick marker and gone over with glued string so they can be felt (lesson 1, and the assessment): strip A split into 4 equal parts with one part marked by a shaded square and a sticker; strip B split into 4 parts of different sizes; strip C split into 3 equal parts.
- A fraction-strip set per pair (lessons 2 to 5): one whole, 2 halves, 3 thirds, 4 quarters, 6 sixths and 8 eighths, all cut from 24 cm card strips so the pieces line up. Write the fraction on every piece in large print and add a raised mark (a glued string line or a puffy-paint dot pattern) for each kind of piece, so colour is not the only clue.
- Three extra whole card strips, 24 cm long, per pair (lesson 2).
- A number-line sheet per pair (lesson 3): four blank lines from 0 to 1, each 24 cm long, and one line from 0 to 2, 48 cm long (two sheets taped end to end), with a large-print or raised-line version. Rulers and pencils.
- A check line per child (lesson 3): a 24 cm line from 0 to 1 with tick marks at each quarter and no labels, in print and raised-line versions. One more check line marked in thirds, for the assessment.
- A short card strip 12 cm long and a long card strip 48 cm long, for the teacher (lesson 4).
- A set of 11 fraction cards per pair, with raised numbers (lesson 4; see lesson 4).
- Large >, < and = cards per pair, with raised outlines (lessons 4 and 5).
- Six story cards for lesson 5 (see lesson 5), each with a small picture.

⚠ Safety and access

- Use counters and cubes too large to swallow. Paper and card only; no scissors are needed, because the strip sets are cut in advance by an adult.
- Raised lines on the check strips, check lines and number lines, and raised marks on every strip piece, let children who are blind or have low vision feel the parts. Laying pieces end to end is a way to compare by touch.
- Folding can be hard for some hands: a partner or adult can fold while the child says where the fold goes, or the child uses pre-marked strips.
- Pointing to digit cards or strips, moving pieces, sign, writing or a communication device all count as answers. Story cards are read aloud, so reading is not what is being checked.

- No public timing, races or times-table tests. The times-table start is a short shared practice, and knowing every fact from memory is an end-of-year aim.
- No real food: the loaves, crackers and cakes in story cards are pretend.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week, with the fraction-strip set, digit cards and the comparison cards to hand:

- Show check strips A and B. "Which strip shows quarters? Why not the other one?" (Strip A: its 4 parts are equal. Strip B has 4 parts, but they are not the same size.)
- "Build 5/6 with your strips. How many sixths make one whole? What is 4/4 the same as?" (5 sixth pieces; 6; one whole.)
- Give the check line in thirds. "Point to where 2/3 goes." (The second tick from 0.)
- "Which is bigger, 3/8 or 3/4? Put the sign between them." (3/4 is bigger: 3/8 < 3/4, because eighths are smaller pieces than quarters.)
- "Use quarter pieces to make a fraction equal to 1/2." (2/4.)
- "What is 4 × 6? What is 8 × 6? How did you work it out?" (24; 48, for example by doubling 24.)

Record yes, with help or not yet for each part. A child who says "3/8 is bigger because 8 is bigger than 4" is using whole-number thinking; note it and come back to laying the pieces side by side.

Sources

- Common Core State Standards for Mathematics, Grade 3 (read to check what is usually expected in grade 3, not to claim alignment), CCSSO PDF: https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf
  - Page 21, grade 3 introduction: the second critical area is "developing understanding of fractions, especially unit fractions (fractions with numerator 1)", and "Students understand that the size of a fractional part is relative to the size of the whole."
  - Page 24, 3.NF.1: "Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b."
  - 3.NF.2: "Understand a fraction as a number on the number line; represent fractions on a number line diagram."
  - 3.NF.3a: "Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line."
  - 3.NF.3b: "Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3."
  - 3.NF.3c: "Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers."
  - 3.NF.3d: "Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole."
  - Page 24, footnote 5: "Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8."
  - Page 26, 3.G.2: "Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole."

    - Page 23, 3.OA.C.7: "By the end of Grade 3, know from memory all products of two one-digit numbers."

Known gaps and cautions

- Times-table facts get only the 5-minute start of each lesson. The 7s and 9s are not practised here, and division facts appear only once (lesson 5), so 3.OA.C.7 still needs daily practice through the rest of the year.
- Splitting shapes into equal areas (3.G.2) is touched on in lessons 1 and 5 with folded paper squares; the year plan gives it more time in term 3, together with area.
- Fractions greater than one go only as far as 8/4 on the 0-to-2 line. Mixed numbers and adding fractions are not taught; they usually come in grade 4.
- Comparing fractions with neither the same top nor the same bottom number is left out, as the standard only asks for the two simpler cases.
- "Top number" and "bottom number" are my choice of words for children; the standards use "numerator" and "denominator", and teachers may introduce those words if they wish.
- The names in story cards (Hen, Goat, Bear, Duck, Mouse, Crab) are the made-up Little Hill characters used in other units. Swap in names and objects that suit your class.
- The lesson order, timings, strip sizes and examples are my choice, not taken from a source.
- Two other agents text-checked this unit on 2026-11-12 and reported no errors: Gemini 3.8 Flash (at 0b7f2b9) and Gemini 4 Argon (at 8c474d7). Those are text checks by AI agents, not a teacher review or classroom test.

Lesson 1: Equal parts and unit fractions

Unit overview - Next: Lesson 2

About 45 minutes.

Goal

Children learn that a fraction needs equal parts of one whole, fold strips into halves, quarters and eighths, mark thirds and sixths, and name one part of each as 1/2, 1/4, 1/8, 1/3 or 1/6.

Materials

- Counters or linking cubes, about 60 per pair.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.
- Plain paper strips 24 cm long and about 4 cm wide, 5 per child, two of them pre-marked with pencil lines in thirds (every 8 cm) and sixths (every 4 cm).
- Paper squares about 15 cm across, 3 per child.
- Three check strips made by the teacher (see the unit Materials): strip A in 4 equal parts with one part shaded and marked with a sticker; strip B in 4 parts of different sizes; strip C in 3 equal parts.

Steps

✎ Opening (5 min)

Times-table start: the 2s. "Two times a number is double the number." Pairs make 3 groups of 2 with counters, then 7 groups of 2 and 9 groups of 2, and show the totals with digit cards (2 × 3 = 6, 2 × 7 = 14, 2 × 9 = 18).

✎ Main activity (32 min)

1. **Is it fair? (8 min).** The teacher folds one paper strip in half, opens it and lays the two parts against each other: the same size. Each part is one half, written 1/2. The bottom number says how many equal parts the whole is split into; the top number says how many of those parts we mean. Now show check strip B. "This strip has 4 parts. Is each part a quarter?" (No: the parts are not the same size.) Then strip A: "Is each part a quarter now?" (Yes: 4 equal parts.)

2. **Fold and name (14 min).** Each child folds and labels their strips, writing the fraction on each part:

    - Fold once: 2 equal parts, each 1/2.
    - Fold in half, then in half again: 4 equal parts, each 1/4.
    - Fold in half three times: 8 equal parts, each 1/8.
    - On the strip marked in thirds, fold along the lines: 3 equal parts, each 1/3.
    - On the strip marked in sixths, fold along the lines: 6 equal parts, each 1/6.

Children line the strips up and say what they notice. (The more parts the whole is split into, the smaller each part is.)

3. **Same square, different quarters (10 min).** Each child folds one paper square into 4 equal parts in three ways: in half and in half again the same way (4 long rectangles); in half one way and then the other way (4 small squares); and corner to corner twice (4 triangles). The parts look different, but in each square every part is 1/4 of the square, because the square is split into 4 equal parts.

✓ Discussion questions (with answers)

- Which is bigger, 1/2 or 1/8 of the same strip? 1/2: the strip is split into fewer parts, so each part is bigger.
- If I cut a strip into 3 pieces of different sizes, is each piece a third? No: thirds have to be 3 equal parts.
- Is a long rectangle quarter the same amount of paper as a triangle quarter of the same square? Yes: each is 1 of 4 equal parts of the same square.

✓ Closing check (8 min, with answers)

Show the three check strips. Each child points to the strip that shows quarters (strip A), names the marked part ("one quarter", 1/4) and names one part of strip C ("one third", 1/3), by saying, signing, pointing to a written fraction or writing it on a whiteboard. Then: "2 × 7?" (14).

Vocabulary

equal parts, whole, fraction, half, third, quarter, sixth, eighth, top number, bottom number

Differentiation

- **Support:** fold halves and quarters only, and compare each part against the whole strip.
- **Stretch:** fold a strip into sixths by first folding in half and then into thirds, and check it against the pre-marked strip.
- Check strips have glued-string lines and a sticker on the marked part, so the parts can be felt. A partner or adult can fold while the child says where the fold goes.

⚠ Safety

Use counters too large to swallow. Paper only; no scissors.

Teacher notes and misconceptions

- Some children call any piece a "half", for example the bigger half. Keep asking "Are the parts equal?"

- Some children think 1/8 is bigger than 1/2 because 8 is bigger than 2. The folded strips are the answer: lay them side by side.
- The standards use "numerator" (top number) and "denominator" (bottom number). Use those words too if your school does.

Lesson 2: Counting in unit fractions

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 45 minutes.

Goal

Children build fractions such as 3/4 from unit fractions ("3 parts of size 1/4"), see that 4/4 makes one whole, and write whole numbers as fractions such as 3 = 3/1.

Materials

- Counters or linking cubes, about 60 per pair.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.
- A fraction-strip set per pair: one whole, 2 halves, 3 thirds, 4 quarters, 6 sixths and 8 eighths, each piece labelled in large print with a raised mark.
- Three extra whole card strips, 24 cm long, per pair.

Steps

✎ Opening (5 min)

Times-table start: the 4s, by doubling the 2s. "2 × 3 = 6, so 4 × 3 is double 6, which is 12." Pairs check with counters, then do 4 × 6 (double 12 is 24) and 4 × 8 (double 16 is 32), showing answers with digit cards.

✎ Main activity (32 min)

1. **Counting quarters (10 min).** The teacher lays the whole strip down and puts quarter pieces on top of it one at a time, and the class counts: "one quarter, two quarters, three quarters, four quarters". Write 1/4, 2/4, 3/4 and 4/4. "3/4 means 3 parts, and each part is 1/4." Four quarters cover the whole strip exactly, so 4/4 = 1.
2. **Build it (12 min).** Pairs build each fraction on top of their whole strip, say how many of which piece they used, and write it:
    - 2/3 (2 third pieces).
    - 5/6 (5 sixth pieces).
    - 3/8 (3 eighth pieces).
    - 6/6 (6 sixth pieces, which cover the whole: 6/6 = 1).
    - 2/2 (2 half pieces, which cover the whole: 2/2 = 1).
3. **Wholes as fractions (10 min).** Pairs lay their three extra whole strips in a row. "How many wholes?" (3.) "If each part is one whole, we have 3 parts of size one whole: 3/1." So 3 = 3/1. Ask: "What would 6/1 be?" (6 wholes, so 6.) "How many eighths make one whole?" (8, so 8/8 = 1.)

✓ Discussion questions (with answers)

- Is 4/4 more than, less than or the same as 1? The same: four quarters cover the whole.
- How many sixths make one whole? 6.
- What does the 3 in 3/8 tell us, and what does the 8 tell us? 3 is how many parts we have; 8 is how many equal parts make the whole.

✓ Closing check (8 min, with answers)

With the fraction-strip set in front of them, each child shows 3/4 (three quarter pieces), answers "How many eighths make one whole?" (8) and "What is 2/2 the same as?" (one whole, 1), and writes "three quarters" as a fraction on a whiteboard or picks the digit cards 3 and 4 (3/4). Then: "4 × 6?" (24).

Vocabulary

unit fraction, whole, three quarters, five sixths, count in quarters

Differentiation

- **Support:** build only with halves and quarters, and count the pieces aloud while touching each one.
- **Stretch:** "Which is more, 7/8 or one whole? How many more eighths would make one whole?" (one whole; 1 more eighth.)
- Raised marks on each piece let children check the piece by touch; pieces can be laid end to end and felt against the whole strip.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- Some children read 3/4 as "three and four" or "three fours". Model the words "three quarters" every time.
- Some children think a fraction must always be less than 1. 4/4, 6/6 and 3/1 show that is not true; lesson 3 puts fractions past 1 on a number line.

Lesson 3: Fractions on a number line

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 45 minutes.

Goal

Children place fractions on a number line by treating the length from 0 to 1 as the whole and marking off equal lengths from 0, go past 1 in quarters, and see that two fractions at the same point are equal.

Materials

- Counters or linking cubes, about 60 per pair.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.
- A fraction-strip set per pair: one whole, 2 halves, 3 thirds, 4 quarters, 6 sixths and 8 eighths, each piece labelled in large print with a raised mark.
- A number-line sheet per pair: four blank lines from 0 to 1, each 24 cm long, and one line from 0 to 2, 48 cm long, in print and raised-line versions.
- Rulers and pencils.
- A check line per child: a 24 cm line from 0 to 1 with tick marks at each quarter and no labels, in print and raised-line versions.

Steps

✎ Opening (5 min)

Times-table start: the 10s and 5s. "5 times a number is half of 10 times it." 10 × 7 = 70, so 5 × 7 = 35; 10 × 4 = 40, so 5 × 4 = 20; then 6 × 5 = 30. Pairs show answers with digit cards and can check 5 × 4 with counters.

✎ Main activity (32 min)

1. **From strip to line (10 min).** The teacher lays the whole strip along a 0-to-1 line: 0 at the start, 1 at the end. "The whole is the distance from 0 to 1." Lay quarter pieces along the line from 0 and make a tick at the end of each one: 1/4, 2/4, 3/4 and 4/4, which lands on 1. "The number 1/4 is the point at the end of the first quarter: it is 1/4 of the way from 0 to 1."

2. **Make your own lines (14 min).** Pairs mark their four 0-to-1 lines by laying pieces from 0 and ticking the end of each piece:

    - Line 1 in halves: 1/2, 2/2.
    - Line 2 in quarters: 1/4, 2/4, 3/4, 4/4.
    - Line 3 in thirds: 1/3, 2/3, 3/3.
    - Line 4 in eighths: 1/8 to 8/8.

Then the 0-to-2 line in quarters: lay the 4 quarter pieces from 0 to 1, tick each end, then move them along to go from 1 to 2. Label 0, 1/4, 2/4, 3/4, 4/4 (also 1), 5/4, 6/4, 7/4, 8/4 (also 2). "5/4 is one quarter more than 1."

3. **Same point (8 min).** Pairs put the halves line under the quarters line with the 0s lined up. "What is at the same point as 1/2?" (2/4.) So 1/2 and 2/4 are equal: the same point on the line. Do the same with the quarters and eighths lines (1/4 and 2/8; 2/4 and 4/8). Also 4/4, 3/3 and 1 are all at the same point.

✓ Discussion questions (with answers)

- Where is 4/4 on the line? At 1.
- Is 5/4 more or less than 1? More: it is one quarter past 1.
- Which is further from 0, 1/3 or 1/8? 1/3, because thirds are longer pieces than eighths.

✓ Closing check (8 min, with answers)

Give each child a check line. The child points to where 3/4 goes (the third tick from 0) and where 1/4 goes (the first tick), then answers "What other fraction is at the same point as 1?" (4/4), by pointing, saying, signing or writing on a whiteboard. Then: "5 × 7?" (35).

Vocabulary

number line, point, tick mark, distance from 0, past 1, equal, same point

Differentiation

- **Support:** mark only halves and quarters, laying each piece from 0 every time.
- **Stretch:** on the 0-to-2 line, where would 3/2 go? (At 6/4, half way between 1 and 2.)
- Raised-line sheets let children feel each line and tick. Laying the strip pieces along the line works by touch as well as by sight.

⚠ Safety

Use counters too large to swallow. Rulers and pencils stay on the desk.

Teacher notes and misconceptions

- Some children write 1/4 at 0. Remind them that 0 is the start and 1/4 is the end of the first piece.
- Some children count tick marks including 0, so they put 3/4 at the second tick. Count the pieces (spaces), not the marks.
- The lines must all be the same length as the whole strip, or the pieces will not fit; this is the "same whole" idea that lesson 4 comes back to.

Lesson 4: Comparing fractions

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 45 minutes.

Goal

Children compare two fractions with the same bottom number or the same top number by reasoning about the size of the pieces, record the result with >, < or =, and explain why the comparison only works for parts of the same whole.

Materials

- Counters or linking cubes, about 60 per pair.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.
- A fraction-strip set per pair: one whole, 2 halves, 3 thirds, 4 quarters, 6 sixths and 8 eighths, each piece labelled in large print with a raised mark.
- Large >, < and = cards per pair, with raised outlines.
- A short card strip 12 cm long and a long card strip 48 cm long, for the teacher.
- A set of 11 fraction cards per pair, in large print with the numbers also raised (glued string or puffy paint): 1/2, 1/3, 1/4, 1/6, 1/8, 2/3, 2/8, 3/4, 3/8, 5/6, 5/8.

Steps

✎ Opening (5 min)

Times-table start: the 3s and 6s. "6 times a number is double 3 times it." 3 × 4 = 12, so 6 × 4 = 24; 3 × 7 = 21, so 6 × 7 = 42. Pairs show answers with digit cards and can build 3 × 4 and 6 × 4 with counters.

✎ Main activity (32 min)

1. **Same bottom number (8 min).** Pairs build 3/8 and 5/8 side by side. "The pieces are the same size, so more pieces is more." 5/8 is bigger than 3/8, written 5/8 > 3/8, and 3/8 < 5/8. Children place the > or < card between the two rows of pieces. Recall the signs from grade 1: the open side faces the bigger number.
2. **Same top number (10 min).** Pairs build 1/3 and 1/6 side by side. "Which piece is longer?" (1/3.) "The bigger the bottom number, the more pieces the whole is cut into, so the smaller each piece is." 1/3 > 1/6. Then 2/3 and 2/8: two pieces each, but thirds are longer, so 2/3 > 2/8.
3. **Same whole? (4 min).** The teacher folds the short strip in half and the long strip into quarters, and holds up one half of the short strip next to one quarter of the long strip. "1/2 is bigger than 1/4, isn't it? So why is this quarter longer?" (The wholes are different sizes: half of 12 cm is 6 cm, a quarter of 48 cm is 12 cm.) We can only compare fractions fairly when they are parts of the same whole.

4. **Compare game (10 min).** The fraction cards go face down. A child turns over two cards. If the two fractions share a top number or a bottom number, the pair builds both with their strips, puts the right sign card between the two fraction cards and says why. If they share neither, put one card back, mix the cards and turn over another. Swap turns.

✓ Discussion questions (with answers)

- Why is 1/6 smaller than 1/3 when 6 is bigger than 3? The whole is cut into 6 pieces instead of 3, so each piece is smaller.
- 3/4 or 3/8: which is bigger, and why? 3/4: both have 3 pieces, and quarters are bigger pieces than eighths.
- Is half always bigger than a quarter? Only when both are parts of the same whole.

✓ Closing check (8 min, with answers)

Each child may use their strips. "Which is bigger, 1/6 or 5/6?" (5/6.) "Put the sign card between 1/4 and 1/8." (1/4 > 1/8.) "Hen eats half of a small pretend cake and Goat eats a quarter of a very big pretend cake. Hen says, 'I ate more, because 1/2 is more than 1/4.' Is Hen sure to be right?" (No: the cakes are different sizes, so we cannot tell without seeing them.) Answers can be said, signed, pointed to or written. Then: "6 × 4?" (24).

Vocabulary

compare, bigger, smaller, greater than, less than, equal to, same whole, same top number, same bottom number

Differentiation

- **Support:** compare only unit fractions (1/2, 1/3, 1/4, 1/6, 1/8), always building both.
- **Stretch:** "Put 1/2, 1/3, 1/4, 1/6 and 1/8 in order from smallest to biggest and explain the pattern." (1/8, 1/6, 1/4, 1/3, 1/2: the bigger the bottom number, the smaller the piece.)
- Fraction cards and sign cards have raised numbers and outlines. Pieces laid side by side from the same starting edge can be compared by touch.

⚠ Safety

Use counters too large to swallow.

Teacher notes and misconceptions

- The most common error is "1/8 is bigger than 1/3 because 8 is bigger than 3". Building both is the answer, every time.
- The 11 fraction cards were chosen so that any allowed pair can be built with one strip set (for example 5/8 and 3/8 need exactly 8 eighths).

- Comparing fractions with different top and bottom numbers, such as 2/3 and 3/4, is left for later grades.

Lesson 5: Equal fractions and fraction stories

Unit overview - Previous: Lesson 4

About 45 minutes.

Goal

Children find simple equal fractions, such as 1/2 = 2/4 = 4/8 and 2/3 = 4/6, by matching lengths, and use fractions to solve short story problems.

Materials

- Counters or linking cubes, about 60 per pair.
- Two sets of digit cards 0 to 9 per pair.
- Whiteboards and pens, or paper and pencils.
- A fraction-strip set per pair: one whole, 2 halves, 3 thirds, 4 quarters, 6 sixths and 8 eighths, each piece labelled in large print with a raised mark.
- Large >, < and = cards per pair, with raised outlines.
- Six story cards (below), each with a small picture.
- Paper squares about 15 cm across, 1 per child.

Steps

✎ Opening (5 min)

Times-table start: the 8s, by doubling the 4s. 4 × 6 = 24, so 8 × 6 = 48; 4 × 7 = 28, so 8 × 7 = 56. Then one division from a fact: "8 × 5 = 40, so what is 40 ÷ 5?" (8.) Pairs show answers with digit cards and can check 8 × 5 with counters.

✎ Main activity (32 min)

1. **Same length, different names (12 min).** Pairs lay one half piece on the table, then find other pieces that make exactly the same length: 2 quarters, then 4 eighths. They put = cards between them and write 1/2 = 2/4 = 4/8. Then:

    - 1/3 = 2/6 (2 sixth pieces match 1 third piece).
    - 2/3 = 4/6.
    - 3/4 = 6/8.

    "Equal fractions are the same size, even though the numbers are different."

2. **Fraction stories (14 min).** The teacher reads each story card aloud. Pairs build it with strips or draw it, then answer:

    - Card 1: Hen cuts a long pretend loaf into 4 equal pieces. Goat eats 1 piece. What fraction of the loaf did Goat eat? How much is left? (1/4; 3/4.)
    - Card 2: Bear's fence has 8 equal parts. Bear paints 5 parts. What fraction is painted? What fraction is not painted yet? (5/8; 3/8.)

    - Card 3: Duck and Mouse have two ribbons the same length. Duck cuts one ribbon into 3 equal pieces, and Mouse cuts the other into 6 equal pieces. Each gives away 1 piece. Who gave away more ribbon? (Duck: 1/3 > 1/6.)
    - Card 4: Crab walks 2/4 of the way along the path. Goat walks 1/2 of the way along the same path. Who walked further? (Neither: 2/4 = 1/2.)
    - Card 5: The school garden has 6 equal beds, and all 6 beds have flowers. What fraction of the garden has flowers? (6/6, which is one whole garden.)
    - Card 6: Mouse has 3 whole pretend crackers. Write 3 as a fraction. (3/1.)
3. **Half of a square (6 min).** Each child folds a paper square in half, then in half again. "The first fold made halves. What are the parts now?" (Quarters.) "How many quarters cover one half?" (2.) So 1/2 of the square = 2/4 of the square.

✓ Discussion questions (with answers)

- Is 1/2 equal to 2/4 even though the numbers are different? Yes: they are the same length, and the same point on the number line.
- How many eighths are the same as 3/4? 6.
- Card 4: why is it fair to compare Crab and Goat? They walk along the same path, so it is the same whole.

✓ Closing check (8 min, with answers)

With the fraction-strip set, each child shows a fraction equal to 1/3 (2/6) and shows 1/2 using eighths (4/8). The teacher then lays 3 quarter pieces on the whole strip: "What fraction of the strip is covered?" (3/4.) Answers can be shown with pieces, said, signed, pointed to or written. Then: "40 ÷ 5?" (8).

Vocabulary

equal fractions, same size, same length, the same as, story problem

Differentiation

- **Support:** find only 1/2 = 2/4, and use cards 1, 4 and 6 with an adult reading each step.
- **Stretch:** find every fraction in the strip set that is equal to 1/2 (2/4, 3/6, 4/8), and explain why there is no number of thirds equal to 1/2. (Thirds do not line up with the half mark: one third is too short and two thirds are too long.)
- Matching lengths works by touch: lay the pieces from the same starting edge and feel whether the ends line up.

⚠ Safety

Use counters too large to swallow. The loaf, crackers and garden are pretend; no real food.

Teacher notes and misconceptions

- Some children think equal fractions must use the same numbers. Keep going back to "Is it the same length?"
- Avoid rules such as "multiply top and bottom by the same number" at this stage; the standards ask children to explain equal fractions with a model.
- Story names are the made-up Little Hill characters used in other units.

Grade 3 Mathematics: Area and Shapes (5 lessons, about 4 weeks)

> **Unreviewed AI draft.** Written by Claude Opus 5.5 (AI Village), 12 November 2026. No educator has reviewed or classroom-tested this unit. It is not aligned to any official standard. Check activities against your school's rules.
>
> Text-checked by four other agents (Gemini 3.5 Flash, Gemini 3.8 Flash, Gemini 4 Argon and Gemini 4 Carbon); see Known gaps.

Back to the grade 3 year plan - Term 3. Builds on grade 2 Measuring, time, money and shapes (measuring in centimetres and metres; naming flat shapes) and grade 3 Equal groups and arrays and Fractions

Big idea

Area is how much flat space a shape covers. We measure it by covering the shape with equal squares, with no gaps and no overlaps, and counting them. When the squares sit in rows, as in a rectangle, we do not need to count them one by one: 4 rows of 6 squares is 4 × 6 = 24 squares. A shape can be split into rectangles and the areas added. Shapes can also be sorted by what they have in common, and a shape can be split into parts with equal areas, each part a fraction of the whole.

Outcomes

By the end of the unit most children can:

1. Explain area as the amount of flat space a shape covers, and measure it by covering the shape with equal squares, with no gaps and no overlaps.
2. Find areas by counting square centimetres on squared paper, and use the words "square centimetres" and "square metres".
3. Find the area of a rectangle by tiling it, explain why it is the same as multiplying the two side lengths, and multiply side lengths to solve story problems.
4. Find the area of a shape made of rectangles (such as an L shape) by splitting it into rectangles and adding their areas, and show that a 6 by 7 rectangle can be split into a 6 by 5 and a 6 by 2 rectangle.
5. Sort four-sided shapes, explain that squares, rectangles and rhombuses are all quadrilaterals, and draw a quadrilateral that is none of these.
6. Split a shape into parts with equal areas and name each part as a unit fraction of the whole, such as 1/4.

Prior knowledge

Grade 2: measuring lengths with a ruler in centimetres, and naming triangles, squares, rectangles, pentagons and hexagons. Grade 3 term 1: arrays, rows and columns, and multiplication as equal groups. Grade 3 term 2: unit fractions as equal parts of one whole. Children who are not yet secure with multiplication can count the squares in rows (skip-counting) in every lesson; the tiles are always there.

Lessons

Each lesson is about 45 minutes: a 5-minute times-table start, paired work with paper squares, tiles or squared paper (32 minutes, ending with a short class discussion), and an 8-minute closing check. Split a lesson over two days if needed; the unit runs over about 4 weeks, so each idea should come back in short daily practice (one rectangle to tile, one area to work out, or one shape to sort).

1. Covering a shape: what is area?
2. Square centimetres and square metres
3. Rows of squares: area and multiplying
4. Splitting shapes and adding areas
5. Sorting four-sided shapes and sharing areas fairly

Materials (whole class unless noted)

- Counters or linking cubes too large to swallow, about 60 per pair (every lesson, for the times-table start).
- Whiteboards and pens, or paper and pencils (every lesson).
- Three card rectangles per pair, cut in advance by an adult and labelled on the back (lesson 1): A, 16 cm by 12 cm; B, 24 cm by 8 cm; C, 20 cm by 12 cm.
- 20 paper squares per pair, each 4 cm by 4 cm, cut in advance (lesson 1).
- 10 paper circles per pair, each 4 cm across, cut in advance (lesson 1).
- Centimetre squared paper, 2 sheets per child (lessons 2 and 4), and a shape sheet per pair with five shapes drawn on centimetre squared paper (lesson 2; see lesson 2).
- One 4 cm paper square from lesson 1 per pair (lesson 2).
- Four metre sticks and masking tape to stick down a 1 m by 1 m square on the floor (lesson 2).
- Square tiles 2 cm across (plastic or card, too large to swallow), about 50 per pair (lessons 3 and 4).
- A set of four rectangle cards per pair, each the right size to be covered by the 2 cm tiles (lesson 3; see lesson 3).
- Six Little Hill garden cards (lesson 3; see lesson 3).
- Coloured pencils, two colours per child (lesson 4).
- An L-shape sheet per pair, drawn on centimetre squared paper with the side lengths written along the edges (lesson 4; see lesson 4).
- A set of twelve shape cards per pair, cut in advance (lesson 5; see lesson 5).
- Two sorting hoops or two large loops of string per pair, and two label cards: "4 square corners" and "4 sides the same length" (lesson 5).
- For the assessment: a card 6 cm by 10 cm, and two check shapes drawn on centimetre squared paper (see Assessment).
- Paper squares 15 cm across, 3 per child, and rulers (lesson 5).
- For blind or low-vision children (all lessons): squares, tiles and circles that can be felt; raised-line squared paper (or a grid of glued string lines) with large-print numbers; shape cards with raised outlines and a Braille or large-print label.

⚠ Safety and access

- Tiles, counters and cubes must be too large to swallow. All cards, squares and circles are cut in advance by an adult, so no scissors are needed.

- The 1 m square in lesson 2 is made with masking tape stuck down flat on the floor, so nobody trips on loose sticks. Metre sticks are carried upright and put down, never waved.
- Nobody measures a person, and nobody's body, desk space or home is used as an example. The stories use made-up Little Hill gardens.
- Raised-line squared paper, tactile tiles and raised shape outlines let children who are blind or have low vision count squares and sides by touch. Laying tiles in rows is a way to "see" a rectangle's area with the hands.
- Placing tiles can be hard for some hands: a partner or adult can place them while the child says where they go, or the child counts squares already drawn on paper.
- Pointing, placing tiles, sign, writing or a communication device all count as answers. Story cards are read aloud, so reading is not what is being checked.
- No public timing, races or times-table tests. The times-table start is a short shared practice.

Assessment

Observe during paired work and keep a class list with one column per outcome. A one-to-one check (about 5 minutes per child) during the fourth week, with tiles, a 4 cm paper square and a 4 cm paper circle from lesson 1, a card 6 cm by 10 cm (3 by 5 tiles), two check shapes drawn on centimetre squared paper (described below), a 15 cm paper square and the shape cards to hand:

- Show a 4 cm paper square and a 4 cm paper circle. "Which shape is better for measuring area by covering? Why?" (The square: squares fit together with no gaps; circles leave gaps.)
- Show a shape on centimetre squared paper made of a 4 by 3 rectangle with one more square joined to one side. "What is its area?" (13 square centimetres.)
- Give a card 3 tiles wide and 5 tiles long. "Cover it with tiles. How many? How could you know without counting every tile?" (15; there are 3 rows of 5, and $3 \times 5 = 15$.)
- "A Little Hill garden is a rectangle 8 metres long and 4 metres wide. What is its area?" (32 square metres.)
- Show an L shape drawn on squared paper: a 5 by 4 rectangle with a 2 by 2 square missing from one corner. "What is its area? Show how you split it." (16 square centimetres, for example $5 \times 2 = 10$ and $3 \times 2 = 6$, and $10 + 6 = 16$.)
- Show shape cards for a square, a rhombus and a kite. "Which of these are quadrilaterals? Is the square a rectangle too?" (All three; yes, a square has four square corners.) Then: "Fold this paper square into 4 parts with equal areas. What fraction is one part?" (1/4.)

Record yes, with help or not yet for each part. A child who counts the squares along the edges of a rectangle and adds them (for example $4 + 3 + 4 + 3$) is measuring the distance round the edge, not the area; note it and come back to covering the shape with tiles.

Sources

- Common Core State Standards for Mathematics, Grade 3 (read to check what is usually expected in grade 3, not to claim alignment), CCSSO PDF: https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf
  - Page 25, 3.MD.5: "Recognize area as an attribute of plane figures and understand concepts of area measurement."

    - 3.MD.5a: "A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area."
    - 3.MD.5b: "A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units."
    - 3.MD.6: "Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units)."
    - 3.MD.7a: "Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths."
    - 3.MD.7b: "Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning."
    - 3.MD.7c: "Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning."
    - 3.MD.7d: "Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems."
    - Page 26, 3.G.1: "Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories."
    - 3.G.2: "Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole."

Known gaps and cautions

- Four other agents (Gemini 3.5 Flash, Gemini 3.8 Flash, Gemini 4 Argon and Gemini 4 Carbon, at ab10d5c) text-checked this unit on 2026-11-12. The two Flash agents and Argon reported no errors; Carbon reported three small items (a quote missing its inner quotation marks, too few tiles in lesson 3, and the lesson 5 label cards missing from this materials list), all fixed. Those are text checks by AI agents, not a teacher review or classroom test. No teacher has reviewed or tried it.
- Perimeter (3.MD.8) is not taught here, apart from one misconception note. The grade 3 year plan does not include it yet; it still needs a unit or short practice later in the year.
- Square inches and square feet are named only in a teacher note; the unit uses square centimetres and square metres. Schools that use inches and feet can swap the squared paper and the floor square.
- The multiplication sign in the 3.MD.7c quote is lost in the CCSSO PDF's text layer; I read it as "a × b and a × c" from the rendered page.
- The times-table start covers the 7s, 9s, 6s and 8s with counters; it does not replace daily practice of all facts.
- "Trapezoid" is the US word; many other countries say "trapezium". The standards do not ask children to name kites or trapezoids; they are on the shape cards only as examples of other quadrilaterals.
- The names in story cards (Hen, Goat, Bear, Duck, Mouse) are the made-up Little Hill characters used in other units. Swap in names that suit your class.
- The lesson order, timings, card sizes and examples are my choice, not taken from a source.

Lesson 1: Covering a shape: what is area?

Unit overview - Next: Lesson 2

About 45 minutes.

Goal

Children compare how much flat space three rectangles cover by covering each one with equal paper squares, find that a long thin rectangle can cover the same space as a shorter, wider one, and learn the word "area".

Materials

- Counters or linking cubes, about 60 per pair.
- Whiteboards and pens, or paper and pencils.
- Three card rectangles per pair (cut in advance by an adult, letter on the back): A, 16 cm by 12 cm; B, 24 cm by 8 cm; C, 20 cm by 12 cm. One large set for the board.
- 20 paper squares per pair, each 4 cm by 4 cm, cut in advance.
- 10 paper circles per pair, each 4 cm across, cut in advance.
- For blind or low-vision children: card rectangles with a raised edge (a glued string line round the outside) and thick card squares and circles that are easy to feel.

Steps

✎ Opening (5 min)

Times-table start: the 7s. Pairs make 3 groups of 7 with counters, then 5 groups of 7, then 7 groups of 7, and write each fact: 3 × 7 = 21, 5 × 7 = 35, 7 × 7 = 49. "How could 5 × 7 help you find 6 × 7?" (Add one more group of 7: 35 + 7 = 42.)

✎ Main activity (32 min)

1. Which covers more? (5 min). Hold up rectangles A and B. "If these were two rugs for the Little Hill library floor, which rug would cover more of the floor?" Pairs talk and write A or B on their whiteboard. Many children choose B because it is longer. Do not give the answer yet.

2. Covering with squares (10 min). Give each pair their rectangles and the 4 cm squares. "Cover rectangle A with squares. The squares must not overlap, and there must be no gaps. Then count them." (12 squares, in 3 rows of 4.) Then cover B. (12 squares, in 2 rows of 6.) "A and B cover the same amount of space, even though B is longer." Now give each pair the circles. "Try covering A with circles. What happens?" (There are gaps between the circles, so some of the card is not covered.) Show on the board what an overlap looks like (one square lying partly on another) and why it counts some space twice.

3. Rectangle C (10 min). Pairs cover C and count. (15 squares, in 3 rows of 5.) They put A, B and C in order from the one that covers the least space to the one that covers the most (A

and B are equal with 12 squares; C is the largest with 15) and write it on the whiteboard. Introduce the word: "The amount of flat space a shape covers is its **area**. The area of A is 12 squares. The area of C is 15 squares."

4. Talk together (7 min). Use the discussion questions below.

✓ Discussion questions (with answers)

- "Was your first guess about A and B right? Why might the longer one look bigger?" (Many people look only at length. Covering shows that A and B have the same area, 12 squares each.)
- "Why are squares better than circles for measuring area?" (Squares fit together with no gaps; circles always leave gaps.)
- "Why must the squares all be the same size?" (If some were bigger, the count would not be fair. It is like measuring length with sticks of different lengths.)
- "Did anyone notice the squares were in rows? How could rows help you count?" (A has 3 rows of 4, so 4 + 4 + 4 = 12. This comes back in lesson 3.)

✓ Closing check (8 min, with answers)

Draw two shapes on the board made of equal squares: shape X is 6 squares in one line; shape Y is 2 rows of 4 squares. Children answer on whiteboards or by pointing:

1. "What is the area of X? Of Y?" (6 squares; 8 squares.)
2. "Which has the larger area?" (Y, even though X is longer.)
3. Show A covered with squares but with one square missing in the middle. "Is this area measured properly? Why not?" (No: there is a gap.)

Vocabulary

- **area**: the amount of flat space a shape covers.
- **gap**: a space that is not covered.
- **overlap**: when one piece lies partly on top of another.

Differentiation

- **Support:** cover only A and B; an adult draws pencil lines on the back of the card in 4 cm squares, so the child can match squares to lines.
- **Stretch:** "Without covering it, how many 4 cm squares would cover a rectangle 12 cm by 8 cm?" (3 rows of 2, so 6 squares.) Then check by covering.
- **Tactile:** raised-edge rectangles and thick squares let children feel when the card is fully covered; running a fingertip over the circles shows the gaps.

⚠ Safety

- All cards, squares and circles are cut in advance by an adult; no scissors. Counters and cubes must be too large to swallow.

Teacher notes and misconceptions

- "Longer means bigger." Rectangle B is there to show that a long thin shape can have the same area as a shorter, wider one.
- Some children overlap squares to "fit" the edge, or leave small gaps. Ask them to slide the squares together until the edges touch.
- The squares here are "our squares" (an improvised unit). Lesson 2 moves to standard square centimetres and square metres.

Lesson 2: Square centimetres and square metres

Unit overview - Previous: Lesson 1 - Next: Lesson 3

About 45 minutes.

Goal

Children learn that a square with sides of 1 centimetre is a **square centimetre**, find the areas of shapes by counting square centimetres on squared paper, draw two different shapes with the same area, and see how big a **square metre** is.

Materials

- Counters or linking cubes, about 60 per pair.
- Whiteboards and pens, or paper and pencils.
- Centimetre squared paper, 1 sheet per child, and pencils.
- A shape sheet per pair (below), drawn on centimetre squared paper.
- One 4 cm paper square from lesson 1 per pair.
- Four metre sticks and masking tape. Before the lesson, an adult lays the sticks in a 1 m by 1 m square on the floor, marks the edges with masking tape stuck down flat, and puts the sticks away.
- For blind or low-vision children: raised-line squared paper (or a grid of glued string lines 1 cm apart) with the shape sheet shapes outlined in thicker raised lines; a thick card 4 cm square.

The shape sheet (each shape is made of whole centimetre squares joined edge to edge; the area is in brackets for the teacher and is not on the sheet):

Shape	What it looks like	Area
P	a rectangle 4 squares long and 3 squares wide	12 square cm
Q	a rectangle 5 squares long and 2 squares wide	10 square cm
R	a row of 4 squares, another row of 4 under it, and 1 more square under the left end	9 square cm
S	a plus sign: 1 square in the middle with 1 square on each of its 4 sides	5 square cm
T	a row of 5 squares with 2 more squares going down from the middle one	7 square cm

Steps

✎ Opening (5 min)

Times-table start: the 9s. Pairs make 2 groups of 9 with counters, then 5 groups of 9, then 6 groups of 9, and write each fact: 2 × 9 = 18, 5 × 9 = 45, 6 × 9 = 54. "How could 10 × 9 help you find 9 × 9?" (Take one group of 9 away: 90 - 9 = 81.)

✎ Main activity (32 min)

1. A square centimetre (5 min). Show one square on the squared paper. "Each side of this square is 1 centimetre long. A square like this is called a **square centimetre**. It is a standard square, the same everywhere, so everyone's measurements can be compared." Pairs put their 4 cm square from lesson 1 on the squared paper, lined up with the lines, and count the square centimetres it covers. (16: 4 rows of 4.)

2. The shape sheet (12 min). Pairs find the area of each shape P to T by counting squares, and write it with its unit, for example "P: 12 square cm". They tick each square lightly with a pencil as they count it, so no square is counted twice. Then: "Which shape has the largest area? Which has the smallest?" (P, 12 square cm; S, 5 square cm.)

3. Same area, different shape (7 min). Each child draws two different shapes on their own squared paper, each with an area of exactly 10 square centimetres. Squares must join edge to edge, not just at a corner. Partners check each other's count.

4. A square metre (4 min). Gather round the taped square on the floor, standing outside the tape. "Each side is 1 metre long, so this is a **square metre**. How many square centimetres do you think would cover it?" (Take a few guesses. The answer is 100 rows of 100, which is 10,000; children do not need to work this out.) "We use square metres for big things like floors and gardens, and square centimetres for small things like a card or a page."

5. Talk together (4 min). Use the discussion questions below.

✓ Discussion questions (with answers)

- "Were everyone's 10-square-centimetre shapes the same shape? Did they have the same area?" (Different shapes; the same area, 10 square cm.)
- "Shape Q is 5 squares long and shape P is only 4 squares long. Which has the larger area?" (P: 12 square cm, more than Q's 10. A longer shape does not always cover more space; we count the squares, not just the length.)
- "Why do we say 'square centimetres' and not just 'centimetres'?" (Centimetres measure length along a line; square centimetres measure the space a shape covers.)

✓ Closing check (8 min, with answers)

Draw on a squared grid on the board a rectangle 3 squares long and 2 squares wide with 1 more square joined to the end of the top row. Children answer on whiteboards or by pointing:

1. "What is the area of this shape?" (7 square cm.)
2. "Would you measure the area of the hall floor in square centimetres or square metres?" (Square metres.)
3. "Mouse says shape S has an area of 9 because the plus sign is 3 squares across and 3 squares down. Is Mouse right?" (No: S covers 5 squares. 3 × 3 = 9 would be right only if the whole 3 by 3 square were filled in; the four corner squares are empty.)

Vocabulary

- **square centimetre**: a square with sides 1 cm long; a unit of area.
- **square metre**: a square with sides 1 m long; a unit of area.
- **unit**: the thing we count when we measure.

Differentiation

- **Support:** start with shapes P and Q only; an adult draws a dot in each square to be counted.
- **Stretch:** draw a shape with an area of 10 square centimetres that is not a rectangle, and a rectangle with an area of 12 square centimetres in two different ways (for example 3 by 4 and 2 by 6).
- **Tactile:** count squares on raised-line paper with a fingertip, putting a small counter on each square once it is counted.

⚠ Safety

- The floor square is marked with masking tape stuck down flat; the metre sticks are put away before children come near it. Children stand outside the tape.

Teacher notes and misconceptions

- Some children count the lines or the corners instead of the squares. Ticking each square as it is counted helps.
- Some children say "centimetres" for area. Ask: "Are you measuring along a line, or the space inside?"
- The written short form cm² (and m²) can be shown if your school uses it; this unit writes the words, "square cm".
- Schools that use inches and feet can use inch squared paper and a 1 foot by 1 foot square on the floor instead of a square metre.

Lesson 3: Rows of squares: area and multiplying

Unit overview - Previous: Lesson 2 - Next: Lesson 4

About 45 minutes.

Goal

Children tile rectangles with square tiles, see the tiles as rows like an array, find that the area is the same as multiplying the two side lengths, and multiply side lengths to find the areas of made-up Little Hill gardens.

Materials

- Counters or linking cubes, about 60 per pair.
- Whiteboards and pens, or paper and pencils.
- Square tiles 2 cm across, about 50 per pair (enough to build Bear's 48-square-metre garden, one tile per square metre).
- A set of four rectangle cards per pair (below), cut in advance by an adult, blank on the front, with the letter on the back.
- Six Little Hill garden cards (below): one large set for the board and one small set per pair, each with a simple drawing of a rectangle with its two side lengths written on the sides.
- For blind or low-vision children: rectangle cards with a raised edge; tactile tiles; garden cards with raised outlines and large-print or Braille numbers.

The rectangle cards (each side is a whole number of 2 cm tiles):

Card	Size	Tiles along each side	Area in tiles
W	6 cm by 8 cm	3 and 4	12
X	10 cm by 4 cm	5 and 2	10
Y	8 cm by 12 cm	4 and 6	24
Z	6 cm by 14 cm	3 and 7	21

The Little Hill garden cards (made up; answers in brackets, not on the cards):

1. Hen's vegetable bed is 7 metres long and 3 metres wide. (21 square metres.)
2. Goat's grass patch is 5 metres long and 6 metres wide. (30 square metres.)
3. Mouse's flower bed is 4 metres long and 4 metres wide. (16 square metres.)
4. Duck's pond deck is 9 metres long and 2 metres wide. (18 square metres.)
5. Bear's berry patch is 8 metres long and 6 metres wide. (48 square metres.)
6. Bear wants a new rectangle patch with an area of 24 square metres. How long and how wide could it be? (Any of 1 by 24, 2 by 12, 3 by 8 or 4 by 6 metres.)

Steps

✎ Opening (5 min)

Times-table start: the 6s. Pairs make 4 groups of 6 with counters, then 6 groups of 6, then 8 groups of 6, and write each fact: 4 × 6 = 24, 6 × 6 = 36, 8 × 6 = 48. "How could 4 × 6 help you find 8 × 6?" (8 is double 4, so double 24: 48.)

✎ Main activity (32 min)

1. Tiling the cards (12 min). Pairs cover each card W, X, Y and Z with tiles, with no gaps and no overlaps, and fill in a table on their whiteboard: card, number of rows, tiles in each row, total tiles. (W: 3 rows of 4, 12; X: 2 rows of 5, 10; Y: 4 rows of 6, 24; Z: 3 rows of 7, 21. Children may turn a card, so "4 rows of 3" for W is also right.)

2. Rows and multiplying (6 min). Ask: "Where have you seen rows like this before?" (Arrays, in term 1.) Write W as 3 × 4 = 12. "The tiles along one side tell us how many rows. The tiles along the other side tell us how many in each row. So we can multiply the two side lengths instead of counting every tile." Check with Y: 4 × 6 = 24. Then: "A card is 5 tiles along one side and 3 tiles along the other. What is its area, without tiling it?" (15 tiles.) Pairs check by building it.

3. Little Hill gardens (10 min). Read the garden cards aloud. Pairs work out each area by multiplying the side lengths, writing the answer with the unit, for example "Hen: 7 × 3 = 21 square metres". Children who are not yet sure of a fact can draw a quick grid or build the rectangle with tiles, one tile for each square metre. Then: "Whose garden has the largest area?" (Bear's berry patch, 48 square metres.) Card 6 is for pairs who finish early; take every answer that multiplies to 24.

4. Talk together (4 min). Use the discussion questions below.

✓ Discussion questions (with answers)

- "Why can we multiply instead of counting every tile?" (The tiles are in equal rows, so it is like an array: rows × tiles in each row.)
- "Goat's patch is 5 by 6. Would a patch 6 by 5 have the same area?" (Yes, 30 square metres; it is the same rectangle turned round.)
- "Bear's new patch could be 4 by 6 or 3 by 8. Do they look the same?" (No, but both have an area of 24 square metres.)

✓ Closing check (8 min, with answers)

Children answer on whiteboards or by pointing:

1. "A card is 6 tiles along one side and 3 tiles along the other. What is its area?" (18 tiles.)
2. Draw a rectangle on the board split into 4 rows of 7 squares. "What multiplication tells us the area? What is it?" (4 × 7 = 28 squares, or 7 × 4.)

3. "A garden is 10 metres long and 5 metres wide. What is its area?" (50 square metres.)

Vocabulary

- **row**: a line of tiles going across.
- **side length**: how long one side of a shape is.
- **square metre**: the unit used for the gardens, from lesson 2.

Differentiation

- **Support:** tile W, X and Y only; say each row aloud as you skip-count (4, 8, 12). Use tiles for the garden cards as well.
- **Stretch:** card 6, then: "Find every rectangle with whole-number sides and an area of 36 square metres." (1 by 36, 2 by 18, 3 by 12, 4 by 9, 6 by 6.)
- **Tactile:** feel each row of tiles and count the rows, then the tiles in one row.

⚠ Safety

- Tiles, counters and cubes must be too large to swallow. Cards are cut in advance by an adult.

Teacher notes and misconceptions

- Some children add the two side lengths (5 + 6 = 11 for Goat). Build the rectangle with tiles and count to show the area is 30.
- Multiplying side lengths works because the squares are the same size as the units along the sides: here, tiles along tile-long sides, or square metres along metre-long sides.
- The garden sizes and names are made up; they are not real data.

Lesson 4: Splitting shapes and adding areas

Unit overview - Previous: Lesson 3 - Next: Lesson 5

About 45 minutes.

Goal

Children split a rectangle into two smaller rectangles and see that the two areas add up to the whole (6 × 7 = 6 × 5 + 6 × 2), then find the areas of L shapes by splitting them into rectangles and adding.

Materials

- Counters or linking cubes, about 60 per pair.
- Whiteboards and pens, or paper and pencils.
- Centimetre squared paper, 1 sheet per child, and pencils.
- Coloured pencils, two colours per child.
- An L-shape sheet per pair (below), drawn on centimetre squared paper with the side lengths written along the edges.
- Square tiles 2 cm across, about 30 per pair, for children who want to build a shape.
- For blind or low-vision children: raised-line squared paper; L shapes cut from thick card with raised grid lines, so the two parts can be felt; tactile tiles.

The L-shape sheet (areas in brackets for the teacher, not on the sheet):

- **Shape 1:** start with a rectangle 6 squares long and 4 squares tall, and take away a block 3 squares long and 2 squares tall from the top right corner. (18 square cm: for example 6 × 2 = 12 along the bottom and 3 × 2 = 6 on top, and 12 + 6 = 18.)
- **Shape 2, the Little Hill park plan:** a rectangle 8 squares long and 5 squares tall, with a rectangle 3 squares long and 2 squares tall joined to the top, at the left end. (46 square cm: 8 × 5 = 40 and 3 × 2 = 6, and 40 + 6 = 46.)
- **Shape 3 (stretch), the steps:** a row of 6 squares, with a row of 4 squares on top of it at the left end, and a row of 2 squares on top of that at the left end. (12 square cm: 6 + 4 + 2.)

Steps

✎ Opening (5 min)

Times-table start: the 8s. Pairs make 3 groups of 8 with counters, then 5 groups of 8, then 7 groups of 8, and write each fact: 3 × 8 = 24, 5 × 8 = 40, 7 × 8 = 56. "How could 5 × 8 and 2 × 8 help you find 7 × 8?" (40 + 16 = 56.)

✎ Main activity (32 min)

1. Splitting a rectangle (8 min). Each child draws a rectangle on squared paper, 6 rows of 7 squares. "What is its area?" (6 × 7 = 42 square cm; children unsure of 6 × 7 can count in

rows.) Now colour the first 5 squares of every row in one colour and the last 2 squares of every row in the other colour. "What is the area of each coloured part?" (6 × 5 = 30 and 6 × 2 = 12.) "What do they add up to?" (42, the same as the whole rectangle.) Write on the board: 6 × 7 = 6 × 5 + 6 × 2.

2. Splitting to help with a hard fact (6 min). Pairs draw a rectangle 4 rows of 9 squares and split it into 4 rows of 5 and 4 rows of 4. (4 × 5 = 20 and 4 × 4 = 16, and 20 + 16 = 36, so 4 × 9 = 36.) "Could you split it a different way?" (Yes, for example 4 × 3 three times: 12 + 12 + 12 = 36.)

3. L shapes (12 min). Give out the L-shape sheet. "These shapes are not rectangles, so we cannot just multiply. But we can split them into rectangles that do not overlap, find each area, and add." Pairs draw a pencil line to split shape 1 into two rectangles, colour each part, and work out each area. (For example 6 × 2 = 12 and 3 × 2 = 6, total 18 square cm. Splitting the other way gives 3 × 4 = 12 and 3 × 2 = 6, also 18.) Then shape 2, the park plan. (40 + 6 = 46 square cm.) Pairs check one shape by counting squares. Shape 3 is for pairs who finish early.

4. Talk together (6 min). Use the discussion questions below.

✓ Discussion questions (with answers)

- "Did everyone split shape 1 the same way? Did everyone get the same area?" (No; yes, 18 square cm. Any split into rectangles that do not overlap gives the same total.)
- "Why must the two parts not overlap?" (The squares in the overlap would be counted twice and the total would be too big.)
- "How does splitting help with a times-table fact you do not know yet?" (Split it into facts you do know and add, like 4 × 9 = 4 × 5 + 4 × 4.)

✓ Closing check (8 min, with answers)

Children answer on whiteboards or by pointing:

1. "A rectangle is 3 rows of 8 squares. Split it into 3 rows of 5 and 3 rows of 3. What are the two areas, and the total?" (15 and 9; 24 square cm.)
2. Draw on a squared grid on the board a rectangle 5 squares long and 3 squares tall with a block 2 squares long and 1 square tall missing from the top right corner. "What is its area?" (13 square cm: for example 5 × 2 = 10 and 3 × 1 = 3.)
3. "Hen cuts her rectangle into two pieces. Is the total area of the pieces bigger, smaller or the same?" (The same.)

Vocabulary

- **split**: cut a shape into parts (here, with a pencil line).
- **L shape**: a shape like the letter L, made of two rectangles.

Differentiation

- **Support:** split only the 6 by 7 rectangle and shape 1; count the squares in each part instead of multiplying.
- **Stretch:** shape 3; then draw your own shape made of two rectangles with an area of exactly 30 square cm.
- **Tactile:** feel the two parts of a card L shape, build each part with tiles if it is small enough, and count the rows.

⚠ Safety

- Tiles, counters and cubes must be too large to swallow. Card L shapes are cut in advance by an adult.

Teacher notes and misconceptions

- Some children add up the side lengths written on the sheet. Ask: "Are you measuring round the edge, or the space inside?" Going round the edge is called the perimeter; it is not part of this unit.
- Some children split an L shape so that the parts overlap and then count the overlap twice. Colouring each part in a different colour shows any overlap.
- The 6 × 7 split is the area model of the distributive property; children do not need the name.

Lesson 5: Sorting four-sided shapes and sharing areas fairly

Unit overview - Previous: Lesson 4

About 45 minutes.

Goal

Children sort shape cards by their sides and corners, learn that squares, rectangles and rhombuses all belong to a bigger group called **quadrilaterals**, draw a quadrilateral that is none of these, and fold paper squares into parts with equal areas, naming each part as a fraction.

Materials

- Counters or linking cubes, about 60 per pair.
- Whiteboards and pens, or paper and pencils.
- A set of twelve shape cards per pair (below), cut in advance by an adult, each shape drawn large with a thick outline and its square corners marked with a small square.
- Two sorting hoops or two large loops of string per pair, and two label cards: "4 square corners" and "4 sides the same length".
- Paper squares 15 cm across, 3 per child, and rulers.
- For blind or low-vision children: shape cards with raised outlines and a raised mark in each square corner; thick paper squares with the fold lines pressed in firmly so they can be felt.

The twelve shape cards (names in brackets for the teacher; the cards have no names on them):

1. A square.
2. A long thin rectangle (not a square).
3. A wide rectangle (not a square).
4. A rhombus that is not a square (four equal sides, no square corners).
5. A parallelogram with two long sides and two short sides and no square corners.
6. A trapezoid (in some countries called a trapezium): one pair of sides going the same way, the other pair not.
7. A kite: two short sides next to each other and two long sides next to each other.
8. A four-sided shape with no equal sides and no square corners.
9. A triangle with a square corner.
10. A triangle with three equal sides.
11. A pentagon.
12. A hexagon.

Steps

✎ Opening (5 min)

Times-table start: mixed facts. "Lesson 4 showed that 6 × 7 = 42. What is 7 × 6?" (42.) Pairs make 4 groups of 9 with counters and say the fact (36), then use it: "What is 9 × 4?" (36.) "What is 8 × 7?" (56; from lesson 4.)

✎ Main activity (32 min)

1. Sorting shape cards (12 min). First sort: pairs put every shape with 4 sides inside one hoop. (Cards 1 to 8 go inside; 9 to 12 stay outside.) "Shapes with 4 sides are called **quadrilaterals**." Second sort: take the 8 quadrilaterals, overlap the two hoops, and put the label cards "4 square corners" and "4 sides the same length" on them. Pairs place each quadrilateral. (Square corners only: cards 2 and 3, the rectangles. Equal sides only: card 4, the rhombus. In both hoops, in the overlap: card 1, the square. In neither hoop: cards 5, 6, 7 and 8.) Name them together: "A shape with 4 square corners is a **rectangle**. A shape with 4 sides the same length is a **rhombus**. The square is in both, so a square is a rectangle and a rhombus too."

2. Draw one that is different (6 min). "Draw a quadrilateral that is not a square, not a rectangle and not a rhombus." Each child draws one on a whiteboard or paper, and a partner checks: 4 sides? Not 4 square corners? Not 4 equal sides? (Any four-sided shape that passes all three checks is right, such as a kite or a trapezoid.)

3. Sharing a square fairly (10 min). Each child folds a paper square into 4 parts with equal areas, then folds a second square into 4 equal parts a different way. (For example: in half and in half again the same way, to make 4 strips; in half one way and then the other, to make 4 small squares; or along both corner-to-corner lines, to make 4 triangles.) "What fraction of the whole square is one part?" (1/4.) Then, on the third square, children use a ruler to mark 3 equal strips 5 cm wide. (Each strip is 1/3 of the square.)

4. Talk together (4 min). Use the discussion questions below.

✓ Discussion questions (with answers)

- "Is a square a rectangle?" (Yes: it has 4 square corners. It is a special rectangle with all sides the same length.)
- "A triangle quarter and a strip quarter look different. Can they both be 1/4?" (Yes: each is one of 4 equal parts of the same square, so each has a quarter of its area. Equal area does not need the same shape.)
- "What do all the shapes in the big hoop have in common?" (4 sides; they are all quadrilaterals.)

✓ Closing check (8 min, with answers)

Children answer on whiteboards or by pointing:

1. Hold up card 4. "Is this a quadrilateral? Is it a rectangle?" (Yes, it has 4 sides; no, it has no square corners.)
2. Hold up card 1. "Name two groups this shape belongs to." (Any two of: quadrilateral, rectangle, rhombus, square.)
3. Draw a square split by lines into 4 parts of different sizes. "Is each part 1/4 of the square?" (No: the parts must have equal areas.)

Vocabulary

- **quadrilateral**: a flat shape with 4 straight sides.
- **rectangle**: a quadrilateral with 4 square corners.
- **rhombus**: a quadrilateral with 4 sides the same length.
- **square corner**: a corner like the corner of a page (a right angle).
- **equal area**: covering the same amount of space.

Differentiation

- **Support:** do the first sort only, and fold one square into 4 equal parts one way; an adult or partner can fold while the child says where the fold goes.
- **Stretch:** "Can you fold or draw a square into 8 parts with equal areas? What fraction is one part?" (1/8.) "Is there a shape that has 4 equal sides but is not a square?" (Yes, a rhombus like card 4.)
- **Tactile:** check square corners by fitting the corner of a page into them; count sides by running a finger round the raised outline.

⚠ Safety

- Shape cards and paper squares are cut in advance by an adult; no scissors. Counters and cubes must be too large to swallow.

Teacher notes and misconceptions

- Many children say "a square is not a rectangle". Go back to the rule: a rectangle has 4 square corners; the square passes that rule.
- A rhombus turned on its point is still a rhombus; turning a shape does not change its group.
- Children do not need the names parallelogram, trapezoid or kite; those cards are there as other quadrilaterals.
- "Square corner" is my choice of word for a right angle; teachers may use "right angle" if the class knows it.

Ratios that keep their meaning

Subject: Mathematics Grade: 6 Lessons: 5 Time per lesson: approximately 50 minutes Status: written (unreviewed) Author: GPT-6 Astra, AI Village, 4 November 2026

Unreviewed AI-authored instructional draft; not classroom-tested or accredited. This is one introductory unit, not a full grade-6 course or a complete K–12 curriculum. Original text and problems: **CC0 1.0**. The reference standards are linked, not republished under this license.

Proposed K–12 mathematics progression · Project status

Big idea

A ratio compares quantities in a stated order. We can change quantities while preserving their relationship, but must keep track of units and the reference whole.

Driving question: How can we change the size of a pattern, purchase, or measurement without changing the relationship it represents?

What students should be able to do

1. Interpret ordered ratio language and distinguish part-to-part from part-to-whole comparisons.
2. Build equivalent-ratio tables and interpret their coordinate plots.
3. Find and interpret unit rates, including a simple fractional unit rate.
4. Use a stated conversion relationship without changing the quantity measured.
5. Interpret percent as per hundred, find a part from a whole, and find a whole from a stated part and percent.
6. Check whether an answer fits the context and explain the relationship.

What students should already know

Multiplication and division with positive whole numbers, simple fractions and decimals, equal partitions, multiplication by 10 and 100, unit labels, and first-quadrant coordinates. The entry check below identifies where to provide support; it is not an admission test.

Lesson list

1. The order and the whole: ratio labels and part-to-whole fractions.
2. Scaling, tables, and coordinates: equivalent ratios, comparison, and plots.
3. One unit, useful units: unit price, constant speed, and measurement conversions.
4. Percent names the reference whole: percent of a quantity and finding the whole.
5. Use the relationship, check the claim: synthesis, seven assessment tasks, explanations, and diagnostic feedback.

Each lesson includes the actual teaching text, worked examples, numbered learner tasks, and corresponding answers. Links lead to substantive lesson pages, not further outline-only indexes.

Materials (whole unit)

Paper, pencils, optional grid paper and 20 counters in two distinguishable labeled types. No food, purchase, internet, or personal device is required. Draw objects if small loose counters are unsuitable. Offer accessible grids and response formats as appropriate; do not rely on color alone.

Assessment

Use the entry check below, lesson exit tasks, and lesson 5's seven-item assessment. Each lesson contains its own answer explanations. Lesson 5 includes a diagnostic table linking errors to earlier teaching, rather than an unvalidated pass threshold. A correct diagram or spoken explanation can demonstrate the same idea as written prose. Record any support provided; do not treat a single timed score as a complete account of learning.

Sources

Topic reference: Common Core State Standards Initiative, Grade 6, Ratios & Proportional Relationships, read 4 November 2026. No external reading is required to complete these lessons.

Reference	Where it is practiced here
6.RP.A.1: ratio concepts and language	Lesson 1, A1
6.RP.A.2: unit rates, non-complex fractions	Lesson 3, A3
6.RP.A.3a: tables, missing values, plots, comparisons	Lesson 2, A2
6.RP.A.3b: unit pricing and constant speed	Lesson 3, A3; speed practice in 3P2
6.RP.A.3c: percent and finding a whole	Lesson 4, A5–A6
6.RP.A.3d: measurement conversion	Lesson 3, A4

This is a topic-to-task map, **not an independent alignment review or evidence of mastery**. The unit uses selected simple contexts; more practice and transfer assessment are needed for a complete course. It does not establish local qualification compliance. Formal slope, algebraic proportionality tests, and percent change are intentionally left for later work.

Known gaps and cautions

No educator review or classroom trial has been completed. The nominal five-session schedule may be too short for learners needing prerequisite work or more practice; extend it. This unit is not a complete grade-6 course, a full standards-alignment review, or an accredited qualification. Formal slope, percentage change, and algebraic proportionality tests are later topics; lesson 5's non-proportional example is optional. Local educators should check mathematical accuracy, access needs, language, pacing, and required standards before adoption.

Teacher implementation notes

Prerequisites: multiplication and division with positive whole numbers; simple fractions and decimals; multiplication by 10 and 100; understanding an equal partition; first-quadrant coordinates. This unit does not teach every prerequisite from scratch. Calculators may support arithmetic after a relationship is represented; they should not replace deciding which quantities to compare.

Materials: paper, pencils, optional grid paper and 20 counters in two distinguishable types. Shapes or tactile markers can replace colors. No food, laboratory materials, purchase, internet access, or personal financial information is required. The tile, printing, and garden contexts are fictional. Display a ratio as words as well as symbols; read tables by giving headers and pairs. Offer large-print grids, tactile plotting, or coordinate descriptions when appropriate. Do not rely on color alone.

Suggested structure: lessons 1–4 each use 5 minutes for retrieval, 12 for explanation/modeling, 10 for supported work, 15 for independent work, and 8 for an exit check and feedback. These are estimates, not deadlines. Lesson 5 uses 10 minutes of synthesis, 25 for an assessment, and 15 for discussion or later feedback. If independent work takes longer, continue next session rather than rushing the final ideas.

Entry check (before lesson 1; about 10 minutes)

- **D1.** Calculate 6 × 4 and 24 ÷ 6. Explain how the two facts connect.
- **D2.** What is 1/4 of 20? Represent it with equal groups or a drawing.
- **D3.** Calculate 15 ÷ 4; a fraction or decimal is acceptable.
- **D4.** Describe where the point (2, 3) is on a first-quadrant coordinate grid.

Use responses diagnostically, not as an admission test. If D1 or D2 is uncertain, rehearse equal-group models before ratio scaling. If D3 is uncertain, revisit sharing 15 into four equal groups before lesson 3. If D4 is uncertain, model horizontal-first, vertical-second plotting before lesson 2; do not interpret an axis-reading error as a ratio error.

Entry-check answer guide

- **D1:** 24 and 4. Six groups of 4 make 24; sharing 24 equally among 6 groups returns 4 per group.
- **D2:** 5. Split 20 into four equal groups and take one. If the learner takes away 4, distinguish “one of four equal parts” from “four fewer.”
- **D3:** 15/4 = 3 3/4 = 3.75. Four groups of 3 use 12; the remaining 3 shared four ways give 3/4 per group. This prepares a fractional unit rate.
- **D4:** From the origin, move 2 units horizontally right and 3 vertically up. The coordinates refer to axis scales, not necessarily two and three physical grid boxes if those boxes use different units.

Lesson 1: The order and the whole

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Interpret and construct a ratio, and explain why “blue to gold” differs from “blue out of all tiles.”

Materials

Paper and pencil; optional counters in two labeled types. Use labeled shapes rather than color alone.

Steps

Success evidence: a learner labels both parts of 2:3, reverses it correctly when asked for gold:blue, and identifies the blue fraction of the total as 2/5.

✎ Retrieve (5 minutes)

1R. Draw two groups of 5 objects. How many objects are there? Write a multiplication statement.

✎ Explain and model (12 minutes)

Draw a small tile pattern as **B B G G G**, saying B means blue and G means gold; shape labels can substitute for colors. Say:

> A ratio compares two quantities in a stated order. Here, for every 2 blue tiles there are 3 gold tiles. Blue to gold is 2 to 3, written 2:3. Gold to blue is 3:2. The labels tell us which comparison we mean; 2:3 alone does not tell us what was counted.

Count all five tiles. Continue:

> Blue to all tiles is 2:5. Blue tiles are 2/5 of the total. The 3 in blue:gold = 2:3 counts only gold tiles; it is not the total. The quotient 2/3 can describe blue tiles per gold tile, but it is not the fraction of all tiles that are blue.

Worked example: A tray contains 4 round and 6 square counters. Round:square = 4:6; square:round = 6:4; round:all = 4:10. Round counters are 4/10 = 2/5 of the total. Group them into two copies of “2 round, 3 square”: the comparison round:square can also be written 2:3. This does not mean there are only 5 counters on the tray; a ratio need not tell the actual total.

Ask learners to point to the quantity counted by each number before calculating. Do not introduce cross-multiplication as a substitute for interpreting the comparison.

✎ Supported investigation (10 minutes)

1G. With counters or a drawing, make three copies of B B G G G. Record blue count, gold count, total, blue:gold, and the blue fraction of all tiles. Explain which quantities changed and which relationship stayed the same. A partner can describe the arrangement instead of moving counters.

If someone writes 6/9 for the fraction of all tiles that are blue, ask “Does the denominator count every tile?” Recount the total; do not merely tell them to add the ratio numbers without connecting that operation to the objects.

✎ Independent practice (15 minutes)

- **1P1.** A box has 6 triangles and 9 circles. Write triangle:circle, circle:triangle, and triangle:all. What fraction of the shapes are triangles?
- **1P2.** A card says “red:green = 3:2.” Could the card describe 12 red and 8 green tiles? Explain using groups.
- **1P3.** Sam says, “Red:green is 3:2, so 3/2 of all tiles are red.” Explain the error and give the red fraction of the total.
- **1P4.** Draw a collection with star:dot = 1:4 and 10 objects in total. Label the counts.

Exit and response (8 minutes)

1E. A collection has 8 striped and 12 plain cards. State striped:plain and the striped fraction of all cards. Explain why the two comparisons have different second numbers.

If the fraction is 8/12, repeat the whole-counting model with smaller counts before lesson 2. If correct but unexplained, ask the learner to label the denominator's referent. Accept an unreduced correct ratio; simplification is not the main goal here.

Vocabulary

Ratio: a comparison of two quantities in a stated order.

Order: which quantity is named first and which second.

Whole: the entire quantity whose parts are being considered.

Differentiation

- **Extra support:** Use labeled counters or drawings and count the whole before writing a fraction; use the specific 1G and 1E feedback in Steps. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.

- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **1R:** 10 objects; 2 × 5 = 10 represents two groups of 5. If a learner writes 5 × 2, ask how their drawing matches their convention rather than rejecting the equal product.
- **1G:** 6 blue, 9 gold, 15 total; blue:gold = 6:9 = 2:3; blue fraction = 6/15 = 2/5. Counts tripled; the blue-to-gold relationship and blue share of the whole did not change.
- **1P1:** Triangle:circle = 6:9 (or 2:3); circle:triangle = 9:6 (or 3:2); triangle:all = 6:15 (or 2:5); fraction triangles = 6/15 = 2/5. Reversed answers need label checks, not just more simplification practice.
- **1P2:** Yes. Four copies of 3 red and 2 green make 12 red and 8 green. Both counts use the same factor, 4.
- **1P3:** The 2 counts green tiles, not all tiles. Each basic group has 3 + 2 = 5 tiles; 3/5 of all tiles are red. A part of this whole cannot exceed the whole, so 3/2 is also an unreasonable red fraction.
- **1P4:** 2 stars and 8 dots; two groups of 1 star and 4 dots give 10 objects. Five objects would meet the ratio but not the specified total.
- **1E:** Striped:plain = 8:12 = 2:3. Striped fraction of all = 8/20 = 2/5. Twelve counts plain cards; 20 counts every card. Simplifying cannot repair choosing the wrong denominator.

Lesson 2: Scaling, tables, and coordinates

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Generate equivalent ratios by multiplying or dividing both quantities by the same positive factor, and connect a table to a coordinate plot.

Materials

Paper and pencil; optional grid paper or an accessible coordinate grid. Use labeled shapes rather than color alone.

Steps

Success evidence: labels axes, gives missing values with a common factor, and rejects adding the same amount as a general equivalence rule.

✎ Retrieve (5 minutes)

2R. In blue:gold = 2:3, what fraction of all tiles is gold? Explain.

✎ Explain and model (12 minutes)

Return to B B G G G. Two copies give 4 blue and 6 gold; three copies give 6 blue and 9 gold. Each new group contains both kinds in the same relationship.

Worked table:

Blue tiles	Gold tiles	Total tiles
2	3	5
4	6	10
6	9	15
10	15	25

Say:

> To turn 2 blue into 10 blue, multiply by 5. The gold count must also multiply by 5, giving 15. Adding 8 to each original count would give 10 blue and 11 gold; that is a different mixture. The required operation preserves equal groups, not equal differences.

Draw a grid. Label horizontal axis **blue tiles**, vertical axis **gold tiles**, and use equal spacing for equal numerical increments on each axis. Plot (2,3), (4,6), (6,9), and (10,15). Read each coordinate pair as a sentence. These points lie on a line through the origin, but we are counting whole tiles: not every point on the continuous line represents an available collection. At (0,0) neither type is present; it is a useful zero-quantity starting point, not a defined ratio 0:0.

An alternative double number line has blue labels 0,2,4,6 and aligned gold labels 0,3,6,9. Put corresponding labels vertically above one another. It represents the same pairs as the table, rather than two unrelated scales.

Supported work (10 minutes)

2G. A paper design uses 3 square pieces for every 4 triangular pieces. Complete square counts 3, 6, 9, 15 with corresponding triangle counts. Plot the four pairs with squares horizontal. Explain the point with horizontal coordinate 9. Use a table-only explanation if the physical plotting format is inaccessible, while still identifying both coordinates.

✎ Independent practice (15 minutes)

- **2P1.** Complete blue:gold = 2:3 pairs: (8, **), (** , 18), (14, __).
- **2P2.** Which pair keeps that same relationship: (6,7), (8,12), or (10,12)? Explain why at least one rejected pair fails.
- **2P3.** Pattern A uses 2 blue for 3 gold. Pattern B uses 3 blue for 5 gold. Compare their blue counts when each has 15 gold tiles. Which has more blue **per gold tile**?
- **2P4.** Lee changes 4:6 to 8:10 by adding 4 to each number. Does this preserve the ratio? Give a counterexample using equal groups, a table, or quotients.

Exit and response (8 minutes)

2E. A pattern has circle:square = 3:5. How many squares go with 12 circles? Describe the common scaling factor and the coordinate pair if circles are horizontal.

If a learner adds 9 to both counts, build four copies of a 3-and-5 group. If only coordinates are reversed, address axis order separately. Do not require formal slope terminology yet.

Vocabulary

Equivalent ratios: comparisons that express the same multiplicative relationship.

Scale factor: the number by which both quantities are multiplied to make another equivalent pair.

Coordinate pair: two numbers giving horizontal and vertical positions, in that order, on labeled axes.

Differentiation

- **Extra support:** Build repeated groups before completing the table; check axis order separately from ratio scaling using the 2E feedback. Offer large-print, tactile, spoken, or

diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.

- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **2R:** 3/5. There are 3 gold tiles out of 2 + 3 = 5 in each basic group.
- **2G:** Pairs (3,4), (6,8), (9,12), (15,20). At horizontal coordinate 9 there are 9 squares and 12 triangles. Every point must use square count first and triangle count second; scale factors from (3,4) are 1, 2, 3, and 5. Plot those points on labeled, consistently scaled axes.
- **2P1:** (8,12), (12,18), (14,21). Common factors from (2,3) are 4, 6, and 7 respectively.
- **2P2:** (8,12) only. Four copies of (2,3) give (8,12). Six blue require 9 gold, not 7; 10 blue require 15 gold, not 12.
- **2P3:** A has 10 blue and B has 9 blue when each has 15 gold. A therefore has more blue per gold tile. Using a common gold count makes the comparison fair; comparing the original blue counts 2 and 3 alone would not.
- **2P4:** No. Doubling 4:6 gives 8:12, not 8:10. Equivalently, 4/6 = 2/3 and 8/10 = 4/5 are different. Adding the same amount keeps a difference, not generally a ratio.
- **2E:** 20 squares; factor 4; coordinate pair (12,20). Twelve is four times 3, so the square count is four times 5. A plotting error with a correct table should be addressed as a representation issue.

Lesson 3: One unit, useful units

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Find a unit rate, state what its units mean, and use a supplied conversion relationship.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Success evidence: chooses the correct division for a requested rate, interprets a fractional answer, and changes a unit without changing the physical quantity.

✎ Retrieve (5 minutes)

3R. Five identical packs contain 30 cards in total. How many cards are in one pack?

✎ Explain and model (12 minutes)

Say:

> A rate relates quantities with their units attached. A unit rate tells how much of one quantity goes with one unit of the other. Decide which unit you want to make "one" before dividing.

Worked example A: Eight notebooks cost 12 units of currency. Cost per notebook = 12 ÷ 8 = **1.5 currency units/notebook**. At this price, 20 notebooks cost 20 × 1.5 = **30 currency units**. This assumes the same price for each notebook and no extra fees. The reverse rate is 8/12 = 2/3 notebook per currency unit; it answers a different question, not a second price.

Worked example B: A model vehicle covers 18 meters in 6 seconds at constant speed. Its speed is 18 ÷ 6 = **3 m/s**. In 10 seconds it covers **30 m** under that assumption. A real journey can have an average speed without moving at that speed every moment; the constant-speed condition is what licenses this prediction.

Worked example C: Use **1 m = 100 cm**. A ribbon is 250 cm long. Groups of 100 cm make meters, so 250 ÷ 100 = **2.5 m**. Conversely, 1.2 m = 1.2 × 100 = **120 cm**. The number gets

smaller when using the larger unit, but the ribbon does not shrink. Keep the unit label beside every answer.

Supported work (10 minutes)

3G. Offer A has 6 identical pencils for 9 currency units; offer B has 10 for 14. Find each price per pencil and decide which has the lower unit price. Then discuss why “lower unit price” does not alone prove which purchase someone should make. No actual purchase is involved.

If a learner divides pencil count by price, ask them to complete the sentence “___ currency units for one pencil” and identify which original quantity is being shared among pencils.

✎ Independent practice (15 minutes)

- **3P1.** A printing service charges 15 currency units for 20 identical pages. What is the cost per page, as a fraction and decimal? What would 8 pages cost at the same rate with no additional charge?
- **3P2.** A cart travels 42 m in 7 s at constant speed. Find its speed and its distance in 12 s at that speed.
- **3P3.** Using 1 m = 100 cm, convert 360 cm to meters and 0.75 m to centimeters.
- **3P4.** Using 1 h = 60 min, convert 2.5 h to minutes. Explain why multiplying, not dividing, is appropriate.

Exit and response (8 minutes)

3E. Nine identical folders cost 18 currency units. Give the unit price and the cost of 7 folders at that price. Then convert a 180 cm shelf length into meters using 1 m = 100 cm.

If a unit price is inverted, return to the sentence frame. If conversion direction is uncertain, compare the size of the two units and estimate before calculating. Correct arithmetic with missing units needs a labeling check, not a wholly new lesson.

Vocabulary

Unit rate: the amount of one quantity for one unit of another.

Constant speed: the same distance traveled in every equal time interval.

Conversion: expressing the same quantity using a different unit.

Differentiation

- **Extra support:** Complete the sentence “currency units for one item” before dividing; estimate the direction of a conversion as in 3E. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **3R:** 6 cards per pack, because 30 ÷ 5 = 6.
- **3G:** A: 9/6 = 1.5 currency units/pencil. B: 14/10 = 1.4 currency units/pencil. B has the lower unit price by 0.1 currency unit/pencil. Purchase needs can also depend on quantity needed, total money available, quality, and unused surplus; no personal household disclosure is needed. If only full packs can be bought, a unit price does not imply one pencil can actually be purchased at that price.
- **3P1:** 15/20 = 3/4 = 0.75 currency unit/page; 8 pages cost 8 × 0.75 = 6 currency units under the stated pricing assumption.
- **3P2:** 42/7 = 6 m/s; 12 × 6 = 72 m. Constant speed supplies the reason the same unit rate applies throughout.
- **3P3:** 360 cm = 3.6 m; 0.75 m = 75 cm. The first divides by 100 centimeters per meter; the second uses 100 centimeters in each meter.
- **3P4:** 150 min. Each hour contains 60 minutes, so 2.5 × 60 = 150. More of the smaller unit is needed to describe the same duration.
- **3E:** 2 currency units/folder; 7 cost 14 currency units. The shelf is 1.8 m long. Do not combine the pricing and measurement values; they are separate questions.

Lesson 4: Percent names the reference whole

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Interpret percent as a rate per 100 and solve both part-from-whole and whole-from-part questions.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Success evidence: identifies the whole before calculating and checks that the resulting part and whole have the stated percent relationship.

✎ Retrieve (5 minutes)

4R. Find 1/2 of 40 and 1/4 of 40. What percentages correspond to these fractions?

✎ Explain and model (12 minutes)

Draw a 10-by-10 grid with 100 cells, or describe 100 equal shares. Say:

> Percent means per hundred. Forty percent of a whole means 40 out of every 100 equal shares of that whole. The whole need not contain exactly 100 objects. A percent tells a relationship, so we must say what the whole is.

Worked example A: Find 30% of 80 cards. Ten percent of 80 is 8; three such parts are **24 cards**. Equivalently, (30/100) × 80 = 24. Check: 24/80 = 3/10 = 30/100. Avoid saying “remove two zeros” as an explanation.

Worked example B: Twelve cards are 25% of a collection. Find the whole collection. Since 25% is 1/4, 12 is one quarter, so the whole is **4 × 12 = 48 cards**. Check: 25% of 48 is 12. Calculating 25% of 12 instead would answer a different question.

Worked example C: Eighteen of 60 cards are striped. What percentage is striped? 18/60 = 3/10 = 30/100, so **30%**. The denominator 60 is the whole collection; 18/(60 − 18) compares striped with plain instead.

Supported work (10 minutes)

4G. A fictional garden diagram has 50 equal squares. Thirty percent are labeled flowers. How many flower squares are there? Separately, another diagram has 14 flower squares, which are 20% of its total. Find that diagram's total. Name the known whole in the first question and the unknown whole in the second.

If students multiply 14 by 0.20, draw five equal parts representing the whole and put 14 in one part. Emphasize that both problems use percent but ask for different missing quantities.

✎ Independent practice (15 minutes)

- **4P1.** Find 15% of 200 stickers. Show a method using 10% and 5%, or a fraction/decimal calculation.
- **4P2.** Eighteen blue counters are 30% of all counters. How many counters are there in total?
- **4P3.** Fifteen of 60 books have a triangle label. What percent have that label?
- **4P4.** A collection has red:blue = 2:3. What percent of all objects are red? Explain why using 2/3 as the red fraction gives the wrong reference whole.

Exit and response (8 minutes)

4E. Eight cards are 20% of a pack. Find the total pack size and check your answer by finding 20% of it.

If a learner finds 1.6, ask which number their calculation treated as the whole. Restore the relationship with five equal groups before introducing any inverse-operation formula. This unit does not yet cover percentage increase/decrease or changing reference wholes in successive percent changes.

Vocabulary

Percent: a rate per hundred.

Part: the amount selected from a stated whole.

Whole: the quantity representing 100% in the comparison.

Differentiation

- **Extra support:** Draw equal groups representing 100% and label which amount is known; follow the 4G and 4E feedback when the part is mistaken for the whole. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **4R:** 20 and 10; 1/2 = 50% and 1/4 = 25%. The percentage names a share, not the count 20 or 10 independently of a whole.
- **4G:** First diagram: 15 flower squares, since 10% of 50 is 5 and 30% is three times that. Second diagram: 70 total squares, since 20% is one fifth and 14 × 5 = 70. The first known whole is 50; in the second, the 100% whole is initially unknown and resolves to 70.
- **4P1:** 30 stickers. Ten percent is 20 and five percent is 10; together they make 15%, or 30. Also 0.15 × 200 = 30.
- **4P2:** 60 counters. Thirty percent is 18, so 10% is 6 and 100% is 60. Check: 0.30 × 60 = 18. Finding 30% of 18 would mistake the part for the whole.
- **4P3:** 25%, since 15/60 = 1/4 = 25/100.
- **4P4:** 40%, because red:all = 2:5 and 2/5 = 40/100. The fraction 2/3 compares red with blue alone; it does not use the whole collection.
- **4E:** 40 cards; 20% = one fifth, so multiply 8 by 5. Check: 0.20 × 40 = 8. The answer must be greater than 8 because 8 is only a part of this pack.

Lesson 5: Use the relationship, check the claim

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Choose a useful representation and solve an unfamiliar combination of ratio, rate, percent, and conversion tasks.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Synthesis (10 minutes)

5S. A design calls for 4 dark tiles for every 5 light tiles. Someone proposes 12 dark and 13 light. Decide whether it preserves the design and show a correction keeping 12 dark. Use objects, a table, or words. Discuss how “both numbers grew by 8” differs from preserving a relationship.

Review four questions learners can ask themselves:

1. What does each number count or measure?
2. What quantity, if any, is the whole?
3. What operation preserves the relationship?
4. Do the size and units of the answer make sense?

Individual assessment (25 minutes, extend as needed)

Diagrams, tables, and short written or spoken explanations are welcome. Record any arithmetic or reading support provided so that interpretation is fair. The teacher key offers diagnostic guidance, not a validated grading scale.

- **A1 — Meaning.** A box has 12 round counters and 18 square counters. Write round:square in simplest whole-number form, and give the fraction of all counters that are round. Explain what each denominator or second number counts.
- **A2 — Scale and plot.** A design uses 4 short strips for every 7 long strips. Complete the pairs (short, long): (8, **), (** , 35), (12, __). Plot the completed points with short strips on the horizontal axis and long strips on the vertical axis. Label axes and a consistent numerical scale.

- **A3 — Unit rate and comparison.** Pack A contains 8 identical sheets and costs 6 currency units. Pack B contains 12 identical sheets and costs 10 currency units. Which has the lower cost per sheet? Show both rates; exact fractions are acceptable.
- **A4 — Conversion.** A paper strip is 2.4 m long. Using 1 m = 100 cm, express its length in centimeters. Explain why the numerical value increases.
- **A5 — Percent of a whole.** A model has 150 tiles, 40% of which are textured. How many are textured? Show a calculation or representation.
- **A6 — Find the whole.** Twenty-one cards are 35% of a collection. How many cards are in the collection? Check the percent relationship.
- **A7 — Evaluate a claim.** “To change 3:5 into an equivalent ratio, add 2 to both numbers, making 5:7.” Is this method valid here? Explain, and give a correct equivalent ratio different from 3:5.

Feedback and transfer (15 minutes or next session)

Return to one problem with a correct answer but unclear explanation, and one problem with a useful representation but arithmetic error. Discuss the mathematics without naming learners or displaying their work without agreement. Invite a revision in a different representation; revision is part of learning, not proof the first answer was dishonest.

5T — Optional extension, not a grade-6 prerequisite: A fictional delivery service charges 3 currency units plus 2 for each package. Costs for 1, 2, and 3 packages are 5, 7, and 9. Does doubling the package count always double the cost? Explain with an example. This previews grade-7 proportionality; it should not become a hidden requirement for passing this unit.

Vocabulary

Reference whole: the quantity treated as the complete amount when stating a fraction or percent.

Unit rate: an amount for one unit of another quantity.

Equivalent ratios: comparisons preserving the same multiplicative relationship.

Differentiation

- **Extra support:** Use the diagnostic table in Teacher notes to select one earlier representation to revisit; allow oral or diagrammatic explanations without changing the relationship being assessed. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

Lesson 5 synthesis and extension

- **5S:** No; 12 dark is three times 4, so it needs 15 light, three times 5. Corrected pair: (12,15). Adding 8 to both gives different multipliers for the two counts.
- **5T:** No. One package costs 5; two cost 7, not 10. Doubling the package count doubles the package-related part of the cost, from 2 to 4 here; the rate stays 2 currency units per package and the fixed charge stays 3. Learners need not use the word “intercept.” This is an extension, not an assessment requirement.

Assessment answer guide

- **A1:** Round:square = **2:3**, simplified from 12:18 by dividing both counts by 6. Round fraction of all = **12/30 = 2/5**. The second ratio number, 3, denotes three equal ratio shares corresponding to the 18 square counters, not three actual counters; the fraction denominator refers to all counters (30 actual, or 5 equal ratio shares). A correct 2:3 with 2/3 for the fraction signals a reference-whole misconception.
- **A2: (8,14), (20,35), (12,21)**. Scale factors from (4,7) are 2, 5, and 3. Plots should have short strips horizontal, long strips vertical, labeled axes, and equally spaced numerical intervals on each axis. Different horizontal and vertical unit scales are allowed; equal numerical increments on the same axis must have equal physical spacing. These three coordinates provide a checkable plot key even without a printed graph.
- **A3: Pack A**. A costs 6/8 = **3/4 = 0.75** currency unit/sheet; B costs 10/12 = **5/6**, approximately **0.8333** currency unit/sheet. Exact comparison: 3/4 = 9/12 < 10/12 = 5/6. A smaller total pack price alone is insufficient reasoning; the pack counts differ.
- **A4: 240 cm**, since 2.4 × 100 = 240. A centimeter is smaller than a meter, so more centimeters count the same strip length. The strip did not physically grow.
- **A5: 60 tiles**. Ten percent of 150 is 15, so 40% is 4 × 15 = 60; alternatively, (40/100) × 150. Textured:all = 60:150 = 2:5 = 40:100 checks the relationship.
- **A6: 60 cards**. If 35% is 21, then 5% is 3 and 100% is 20 × 3 = 60. Alternatively, 21 ÷ 0.35 = 60. Check: 0.35 × 60 = 21. Multiplying 21 by 0.35 incorrectly treats the given part as the whole.
- **A7: Not valid here.** The proposed ratio 5:7 does not equal 3:5; for example, multiplying 3 by 5/3 would require multiplying 5 by 5/3, yielding 25/3 rather than 7. A whole-number comparison is to scale 3:5 to **21:35** and 5:7 to **25:35**: the same second quantity goes with different first quantities; or use 3/5 = 0.6 and 5/7 ≈ 0.714. A correct equivalent ratio is **6:10** by doubling both original counts. The intended accessible explanation is that changing 3 to 5 is not the same factor as changing 5 to 7; a diagram can show the mismatch.

Diagnostic recording, not a validated pass score

For each outcome record **demonstrated**, **partial evidence**, or **needs another check**, with a short example from the student's work. “Demonstrated” requires the relevant computation or

representation plus a sound interpretation; it does not require the wording above. Avoid combining these categories into a pseudo-precise mastery percentage.

Evidence	If uncertain, revisit
A1 ratio order and reference whole	Lesson 1: count and label each set
A2 scaling and plot; A7 equivalence explanation	Lesson 2: copies of a basic group; coordinates separately
A3 cost per one sheet	Lesson 3: decide the denominator unit before dividing
A4 conversion	Lesson 3: same length, different-sized units
A5 known whole; A6 unknown whole	Lesson 4: identify and draw the whole before calculating

Keep A2's table and graph observations separate. A learner who cannot physically draw a graph may demonstrate the same coordinate relationship using a tactile grid or precise coordinate placement instructions. If reading, language, or arithmetic support was provided, record it factually rather than assuming the mathematical reasoning is absent. Re-teach the specific gap, offer a fresh analogous task, and check again. Classroom educators must choose reporting and progression policies; this draft supplies no normed test, diagnosis, or qualification decision.

Proportional relationships: what stays constant?

Subject: Mathematics Grade: 7 Lessons: 5 Time per lesson: about 60 minutes

Status: written (unreviewed AI draft; not human-educator-reviewed or classroom-tested)

Author: GPT-6 Astra

Big idea

In a proportional relationship, multiplying one quantity by a factor multiplies the other by the same factor. Its equation is y = kx: the rate y/x stays constant for positive x. A percentage calculation also needs a clearly identified reference whole, which can change from one step to the next.

What students should be able to do

- Compute a unit rate from fractional quantities, label its units, and distinguish it from the inverse rate.
- Distinguish proportional from nonproportional relationships in tables, verbal rules and graphs.
- Write y = kx and explain the constant k and points (0, 0) and (1, k), using the quantities and units on the axes.
- Calculate percent increase or decrease and successive percentages with the correct reference whole at each step.
- Find an original amount from a stated final percentage and explain a simple-interest model.

What students should already know

Students need whole-number and fractional unit rates, percent of a whole, fraction multiplication and division, decimal operations, and nonnegative coordinate pairs. The grade 6 ratios unit supplies the ratio and percent foundation; this is not a replacement for instruction in fraction division.

Use these three diagnostic questions without a time limit. Ask for a representation or explanation as well as a result:

1. Three identical notebooks cost 6 currency units, with no fee. What is the cost per notebook?
2. Find 25% of 80.
3. How many halves are in three quarters? Calculate (3/4) ÷ (1/2).

Diagnostic guide: (1) 6 ÷ 3 = 2 currency units per notebook. (2) One quarter of 80 is 20. (3) Three quarters contains one and a half halves, so the quotient is 3/2. A strip divided into quarters makes the comparison visible. These questions are not an admission gate: revisit equal groups, a 100% bar, or fraction strips where needed before combining the ideas.

Lesson list

1. A rate when both quantities are fractions: compute and interpret rates, including inverse units.
2. A straight line is not enough: connect rules, tables, graphs and the origin.
3. Percent change needs a starting whole: distinguish change from final amount and identify the denominator.
4. Two percentages, two reference wholes: successive changes, reverse percent and fees.
5. Choose the model, explain the result: simple interest, synthesis and an eight-task assessment.

Materials (whole unit)

Paper, pencil, ruler, and optional grid paper or tactile grid; fraction strips can be drawn rather than purchased. Offer large-print pages and verbal descriptions. A calculator is optional after a student has written or described the intended calculation. No internet, purchases, food preparation or personal financial information is required.

All currency amounts and rates are fictional arithmetic examples. Substitute a locally familiar currency label if helpful, without changing the stated mathematical rule. These are not actual tax rules or financial advice. Counts of indivisible packs use whole-number inputs; lengths, times and volumes may use fractional inputs.

Assessment

Use lesson 5's A1–A8 tasks and worked guide. Accept oral explanations, labelled drawings or equations; do not score speed or handwriting. Look separately for rate direction and units, proportional-model recognition, graph interpretation, and correct percent bases. Use errors to select a worked example for revisiting, then offer a similar new question. There is no validated pass threshold or claim that eight tasks measure all grade 7 mathematics.

Sources

The Common Core grade 7 Ratios and Proportional Relationships reference was consulted on November 4, 2026. 7.RP.A.1 informed fractional unit rates; 7.RP.A.2a–d informed tables, graphs, equations and point interpretation; 7.RP.A.3 informed multi-step ratio and percent problems. This is a scope cross-reference, not accreditation, a complete alignment audit, or a full grade 7 course. Examples and explanations here are original CC0 material; the linked reference is not relicensed by this repository.

See also the proposed mathematics progression.

Known gaps and cautions

Pacing is an untested estimate. Split lessons, especially lessons 4 and 5, if explanation or practice needs more time. A qualified local educator should check language, access needs, prerequisites, curriculum requirements and examples before classroom use. An agent peer check is not human educator review.

The unit uses nonnegative quantities and does not develop general functions on negative inputs. It is not a finance course. Compound interest appears only as an optional contrast; percent error, commission and other contexts are not exhaustively covered. Data agreeing with a model at

several points does not prove that it holds at unobserved values. Explicit rules in the exercises are assumptions, not empirical claims about equipment, motion or prices.

Lesson 1: A rate when both quantities are fractions

Time: about 60 minutes (untested estimate; split if needed)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Compute a unit rate from fractional quantities and explain why reversing the division changes both the rate and its units.

Materials

Paper, pencil, optional fraction strips and calculator. Ribbon is a fictional measured quantity: no cutting or purchases are needed.

Steps

✎ 1. Opening — 5 minutes

O. In a fictional shop, 3 metres of ribbon cost 6 currency units, with a constant price per metre and no fee. Find the price per metre. Ask students to name the quantity that should become **one**, rather than starting with a memorized division rule.

✎ 2. Main activity: worked model — 15 minutes

Display: “Three quarters of a metre cost nine eighths of a currency unit, at a constant price per metre.” Write the question in words: **How much currency for one metre?**

Draw a one-metre strip divided into four quarters. Label three quarters together 9/8 currency units. One quarter costs (9/8) ÷ 3 = 3/8. Four quarters cost 4 × 3/8 = 12/8 = 3/2 currency units.

Length (metres)	Cost (currency units)
0	0
1/4	3/8
3/4	9/8
1	3/2

Connect the strip to division: (9/8) ÷ (3/4) = (9/8) × (4/3) = 3/2 **currency units per metre**. Multiplying both the length and the cost by 4/3 turns 3/4 metre into one metre. The fraction calculation and the strip describe the same scaling.

Write cost = (3/2) × length, when length is measured in metres. For continuous ribbon sold by length, fractional metre inputs are allowed in this model.

Now reverse the question: “How many metres for one currency unit?” The calculation is (3/4) ÷ (9/8) = 2/3 **metre per currency unit**. Both rates are correct, but they answer different questions. Do not drop the units when comparing them.

3. Guided work — 10 minutes

G. A constant-motion model travels 5/6 kilometre in 1/3 hour. Find kilometres per hour and the distance in 1/2 hour. Have students first scale the time to one hour, then use the rate for half an hour. A single journey's average rate does not by itself establish that the speed stays constant; here constancy is part of the stated model.

✎ 4. Independent practice — 20 minutes

For each task, write the division with units or describe the scaling. A calculator may check the arithmetic after the model is chosen.

- **P1.** A constant-price ribbon model gives 2/3 metre for 5/6 currency unit. Find currency units per metre and the cost of 2 metres.
- **P2.** In a constant-flow model, 3/5 litre fills in 2/5 minute. Find litres per minute and the time needed for 2 litres.
- **P3.** A length of 3/8 metre costs 3/4 currency unit. Find both currency units per metre and metres per currency unit.
- **P4.** Someone divides 3/4 metre by 9/8 currency units and reports “2/3 currency unit per metre.” Explain the mistake and give the price per metre.

5. Discussion — 5 minutes

D. Why can two different numbers both be correct unit rates for the same purchase? Ask for the question each answers, not just the phrase “flip it.”

✓ 6. Closing check — 5 minutes

E. A constant-motion model travels 7/10 kilometre in 1/5 hour. Find kilometres per hour and the distance in 2/5 hour.

Answer guide

- **O:** 6 ÷ 3 = 2 currency units per metre; multiplying 2 by 3 metres gives the original cost 6.
- **G:** (5/6) ÷ (1/3) = (5/6) × 3 = 5/2 kilometres per hour. Half an hour gives (5/2) × (1/2) = 5/4 kilometres. Alternatively, half an hour is 3/2 times the given time, so multiply the given distance by 3/2 too.
- **P1:** (5/6) ÷ (2/3) = 5/4 currency units per metre. Two metres cost (5/4) × 2 = 5/2 currency units.
- **P2:** (3/5) ÷ (2/5) = 3/2 litres per minute. Time = 2 ÷ (3/2) = 4/3 minutes. Check: (3/2) × (4/3) = 2 litres.
- **P3:** (3/4) ÷ (3/8) = 2 currency units per metre; (3/8) ÷ (3/4) = 1/2 metre per currency unit. The reciprocal has different units.
- **P4:** The division produces 2/3 **metre per currency unit**, not currency per metre. For the requested price, (9/8) ÷ (3/4) = 3/2 currency units per metre.

- **D:** “One rate tells me the cost of one metre; the other tells me the length bought for one currency unit. I must say which one I want.”
- **E:** (7/10) ÷ (1/5) = 7/2 kilometres per hour. In 2/5 hour the distance is (7/2) × (2/5) = 7/5 kilometres. This is twice the original distance because the time doubled.

Vocabulary

- **Unit rate:** amount of one quantity for one unit of another, with units specified.
- **Inverse rate:** the rate for the reversed question; for a nonzero rate its numerical value is the reciprocal when the units are reversed consistently.
- **Constant rate:** a rate that does not change within the stated model.

Differentiation

- **Extra support:** use the quarter-metre strip before symbolic fraction division. Read “currency divided by metres” aloud. Allow an oral answer with a labelled strip.
- **Extra challenge:** express the worked ribbon cost as y = kx. **Guide:** x is metres, y is currency units, k = 3/2 currency units per metre. Fractional nonnegative x values are meaningful for this continuous-length model.

⚠ Safety

Use paper models. Do not enact speed or liquid-flow experiments. No purchases or disclosure of personal spending are needed.

Teacher notes

A correct number with reversed units is not yet a correct interpretation. Ask “one of what?” before selecting the division. Fraction strips explain the scaling; a rule for dividing fractions should not replace that explanation. Keep the constant-rate assumption explicit when moving from an observed pair to another time or length.

Unit overview · Next lesson

Lesson 2: A straight line is not enough

Time: about 60 minutes (untested estimate)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Recognize a proportional relationship and connect its constant rate, equation and graph, including the meaning of the origin and the unit-rate point.

Materials

Paper, ruler, optional grid or tactile grid and calculator. Graphs may be described orally with labelled coordinates.

Steps

✎ 1. Opening — 5 minutes

O. Four identical cards cost 6 currency units, with a constant price per card and no fee. What does one card cost? What do eight cost? Name the assumption that permits the second calculation.

✎ 2. Main activity: worked comparison — 15 minutes

Two fictional equipment-use models relate hours x to cost y. Model A charges 3 currency units per hour with no fee: y = 3x. Model B charges a fixed access fee of 2 currency units plus 3 per hour: y = 2 + 3x. The access fee applies even at zero usage once access is purchased.

Hours x	A: cost y	B: cost y
0	0	2
1	3	5
2	6	8
4	12	14

For positive x, A has y/x = 3 at every listed point. B has ratios 5/1 = 5, 8/2 = 4, and 14/4 = 3.5. Its cost per hour of use is not constant even though the charge **for each additional hour** is constant.

Draw horizontal hours from 0 to 4 and vertical currency units from 0 to 14. Use equal spacing for equal numerical increments on each axis; the two axes need not use the same physical scale. Label the axes and plot both sets. Join each model's points with a straight line over nonnegative hours, since this example allows fractional hours and explicitly gives a linear rule. Only A's line passes through the origin.

Explain A's points: (0, 0) means no hours and no cost; (1, 3) means one hour costs 3 currency units, so k = 3 currency units per hour; (2, 6) means two hours cost 6. We do **not** calculate 0/0 at the origin. The rate is determined at positive x, and y = kx gives zero output at zero input.

A straight line alone is insufficient: B is straight. Passing through the origin alone is also insufficient: a curved or irregular relation can include (0, 0). A proportional graph has both properties, over its allowed domain. For discrete counts, the allowable points lie on such a line, but intermediate points may not be valid quantities.

3. Guided work — 10 minutes

G. Ribbon cost y in currency units is y = 2.5x, where x is length in metres. Plot or describe (0, 0), (1, 2.5), (4, 10) and find the point for 3 metres. Explain k and the unit-rate point.

✎ 4. Independent practice — 20 minutes

- **P1.** A table has x values 0, 2, 4, 6 metres and y values 0, 5, 10, 15 currency units. Does it fit a proportional model? State k and its equation. **Assuming that model continues**, find y when x = 10.
- **P2.** A table has x values 0, 1, 2, 3 and y values 0, 2, 5, 9. Does it fit a proportional model? Explain why the origin does not settle the question.
- **P3.** For length x in metres and cost y in currency units, compare y = 4x with y = 4 + x. Which is proportional? For the proportional model, explain (1, 4).
- **P4.** A stated proportional relationship includes (2, 7), with x in hours and y in currency units. Find k, the equation, the unit-rate point and the point for zero hours.

5. Discussion — 5 minutes

D. Does agreement with a proportional model at three measured positive inputs prove that all future measurements will fit it? Distinguish a pattern in data from an explicitly assumed rule.

✓ 6. Closing check — 5 minutes

E. Identical whole packs have x values 0, 3, 6 and costs y of 0, 12, 24 currency units, with a constant price per pack and no fee. Find k. Explain why (1, 4) is a valid purchase but (1/2, 2) is not allowed when only whole packs are sold, even though both lie on y = 4x.

Answer guide

- **O:** 6/4 = 1.5 currency units per card; eight cost 8 × 1.5 = 12. The price per card remains constant and no fixed fee is added.
- **G:** k = 2.5 currency units per metre. (1, 2.5) represents one metre costing 2.5 currency units. At 3 metres, y = 2.5 × 3 = 7.5, so the point is (3, 7.5). The stated rule gives a straight line through (0, 0).
- **P1:** Yes: 5/2 = 10/4 = 15/6 = 2.5 currency units per metre, and zero maps to zero. The fitted model is y = 2.5x; under the continuation assumption, y = 25 currency units at 10 metres.

- **P2:** No: the positive-input ratios are 2, 2.5 and 3, not one constant. Having (0, 0) does not fix this mismatch.
- **P3:** y = 4x is proportional, with k = 4 currency units per metre. (1, 4) means one metre costs 4 currency units. y = 4 + x is not proportional: zero length has cost 4, and the positive-input ratios differ.
- **P4:** k = 7/2 = 3.5 currency units per hour; y = 3.5x; unit-rate point (1, 3.5); zero-hours point (0, 0). No division by zero is used.
- **D:** “The measurements fit the model so far, but an unmeasured value might not. If the exercise gives the rule for the whole domain, I can calculate from that assumption.”
- **E:** k = 12/3 = 24/6 = 4 currency units per pack. One whole pack costs 4. The half-pack point is on the mathematical line but outside the allowed whole-pack inputs in this situation.

Vocabulary

- **Constant of proportionality k:** the fixed multiplier in y = kx; y/x for positive x in these examples.
- **Origin:** the point (0, 0).
- **Domain:** the allowed input values in a model.
- **Linear but nonproportional:** a straight-line relationship with a nonzero intercept, such as y = 2 + 3x.

Differentiation

- **Extra support:** say each coordinate as “horizontal quantity, then vertical quantity.” Use a tactile grid, enlarged ticks or oral descriptions. Compare A and B at x = 0 before computing rates.
- **Extra challenge:** reverse A's axes. **Guide:** hours = (1/3) × cost, so the reversed constant is 1/3 hour per currency unit, the reciprocal of 3 currency units per hour.

⚠ Safety

This is a paper activity; no equipment hire or other purchase is needed.

Teacher notes

Equal increases in y for equal increases in x establish a straight-line pattern in a table, not necessarily proportionality. Avoid calling y = 2 + 3x proportional because its “rate” is 3: its slope is 3, but its total y/x is not constant. A finite table is evidence of fit, not proof about unobserved data. For discrete contexts, draw or describe allowable points separately from the containing line.

Previous lesson · Unit overview · Next lesson

Lesson 3: Percent change needs a starting whole

Time: about 60 minutes (untested estimate)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Calculate percent increase or decrease using the original amount as the reference whole, and distinguish the change from the final amount.

Materials

Paper, pencil, optional calculator and drawn or tactile percentage bar.

Steps

✎ 1. Opening — 5 minutes

O. Find 30% of 50. Name the whole and say whether your answer is the part or the whole.

✎ 2. Main activity: worked comparison — 15 minutes

Display: “A collection grows from 80 counters to 100 counters.” The increase is 100 − 80 = 20 counters. Percent increase compares that change to the **starting** 80:

percent increase = (20/80) × 100% = 25%.

Draw a bar for 80 counters, labelled 100%. Divide it into four equal parts: each represents 20 counters, or 25% of the original. Add one more such part. The final 100 counters represent 125% of the original 80, not 25% of it.

Connect the representations:

- Increase: 0.25 × 80 = 20 counters.
- Final amount: 80 + 20 = 100 counters.
- Equivalent multiplier: 1.25 × 80 = 100 counters.

Now reverse the journey: “A collection shrinks from 100 counters to 80.” The decrease is still 20 counters, but the starting whole is now 100. Percent decrease = (20/100) × 100% = 20%. Draw a fresh bar labelled 100 counters = 100%; do not reuse the old 100% label on 80.

For a percent change written as a decimal r, an increase gives final amount = original × (1 + r); a decrease gives final amount = original × (1 − r). These percent-change calculations require a positive original amount. Here decreases have $0 \le r \le 1$ so the final amount is

nonnegative. A 100% decrease leaves zero; an increase can exceed 100%. Multipliers describe the final amount, not just the change.

3. Guided work — 10 minutes

G. A length changes from 60 centimetres to 45 centimetres. Find the decrease in centimetres, the percent decrease, and the final length as a percentage of the original. Ask students to label 100% before calculating.

✎ 4. Independent practice — 20 minutes

For each answer, name the original whole and include a checking calculation.

- **P1.** A count grows from 50 to 65. Find the percent increase.
- **P2.** A length shrinks from 120 centimetres to 90 centimetres. Find the percent decrease.
- **P3.** Increase 30 counters by 40%. Find the increase and the final count; explain why the two answers differ.
- **P4.** Someone says, “80 to 100 is a 20% increase because 20/100 = 0.20.” Identify the wrong reference whole. What is the correct percent increase, and what is the percent decrease for 100 to 80?

5. Discussion — 5 minutes

D. Why do the forward and backward changes between 80 and 100 have different percentages even though both involve 20 counters?

✓ 6. Closing check — 5 minutes

E. A count grows from 200 to 230. Find the percent increase. Complete: “230 is ___% of 200.”

Answer guide

- **O:** 0.30 × 50 = 15. The whole is 50; 15 is the 30% part. It is not a new total unless a problem specifies how that part is used.
- **G:** Decrease = 60 − 45 = 15 centimetres. Percent decrease = (15/60) × 100% = 25%. Final length is 75% of the original: 0.75 × 60 = 45 centimetres.
- **P1:** Increase = 65 − 50 = 15; (15/50) × 100% = 30%. Check: 1.30 × 50 = 65.
- **P2:** Decrease = 120 − 90 = 30 centimetres; (30/120) × 100% = 25%. Check: 0.75 × 120 = 90 centimetres.
- **P3:** Increase = 0.40 × 30 = 12 counters. Final count = 30 + 12 = 42 counters, or 1.40 × 30. The increase alone omits the original 30 counters.
- **P4:** The increase starts at 80, not 100: 20/80 = 0.25, so the increase is 25%. The reverse starts at 100: 20/100 = 0.20, so the decrease is 20%.
- **D:** “The numerator, 20 counters, is the same, but the denominator is the starting whole. Twenty is a larger fraction of 80 than of 100.”
- **E:** Increase = 30; (30/200) × 100% = 15%. The final 230 is 115% of 200 because 1.15 × 200 = 230.

Vocabulary

- **Reference whole:** the amount that represents 100% in a calculation.
- **Percent increase/decrease:** the increase/decrease divided by the original positive amount, expressed as a percentage.
- **Multiplier:** the factor applied to the original amount to obtain the final amount.

Differentiation

- **Extra support:** draw and label the starting 100% bar before finding any part. Use “starting,” “change” and “final” as three separate labels. Offer oral explanations or a calculator once the reference whole is chosen.
- **Extra challenge:** increase 12 by 150%. **Guide:** the increase is $1.5 \times 12 = 18$; the final amount is $12 + 18 = 30$, or 2.5×12. An increase larger than the original amount is possible.

⚠ Safety

Use fictional counts and lengths, not body measurements, personal income or comparisons between students.

Teacher notes

A final amount and a change are different quantities. “25% increase” means 125% remains relative to the original; “25% decrease” means 75% remains. Do not assume the same numerical change implies the same percentage when the starting whole changes. Percent change from an original amount of zero is undefined in this formula, not automatically 0% or 100%.

Previous lesson · Unit overview · Next lesson

Lesson 4: Two percentages, two reference wholes

Time: about 60 minutes (untested estimate; consider two sessions)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Solve successive and reverse percentage problems by identifying the reference whole at each step.

Materials

Paper, pencil and optional calculator. All prices, taxes and fees are invented for arithmetic, not statements of current policy or financial advice.

Steps

✎ 1. Opening — 5 minutes

O. Find 10% of 90. Explain what amount represents 100%.

✎ 2. Main activity: worked model — 15 minutes

A fictional item costs 80 currency units. Its price is discounted by 25%. A fictional 10% tax is then calculated **on the discounted price**. Find the total.

Write a label for the base at each stage:

Stage	Reference whole	Calculation
Discount	80	0.25 × 80 = 20
Price after discount	80	80 − 20 = 60
Tax	60	0.10 × 60 = 6
Total	60	60 + 6 = 66

The two multipliers give the same result: 80 × 0.75 = 60, then 60 × 1.10 = 66 currency units. Do not calculate 80 × (1 − 0.25 + 0.10) = 68: that incorrectly uses 80 as the base for both percentages. The problem expressly makes 60 the tax base.

Use a bar labelled 80 = 100%, with a quarter removed. Relabel the remaining 60 as 100% **for the tax step**, then add a tenth of that new whole. The quantity did not change when its percentage label changed; the reference did.

Explain a reverse problem: if a final price is 72 after a 20% markdown, the final amount is 80% of the unknown original x. Thus 0.80x = 72 and x = 72 ÷ 0.80 = 90. Check forward: 90

× 0.80 = 72. Adding 20% of 72 is not the inverse operation, because that uses the wrong base.

3. Guided work — 10 minutes

G. A length of 50 centimetres increases by 20%, then decreases by 20% of its **new** length. Find both intermediate and final lengths and the net percent change from the original. Before calculation, ask whether the two changes must cancel; return to that prediction after calculating.

✎ 4. Independent practice — 20 minutes

Use the stated bases, not assumptions about real shops. Write the expression before using a calculator. In these fictional problems, round monetary results to the nearest 0.01 currency unit only at the end; the supplied examples already terminate exactly.

- **P1.** A price of 120 currency units has a 15% discount, followed by a 10% tax on the discounted price. Find discounted price, tax and total.
- **P2.** A count of 200 grows by 10%, then by another 10% of the new count. Find the final count and overall percent increase from 200.
- **P3.** A final price is 72 currency units after a 20% markdown. Find the original and check it. Explain why 72 × 1.20 does not recover it.
- **P4.** Policy A adds a 10% tax on an 80-currency-unit item, then adds a flat fee of 5. Policy B taxes the combined item and fee, 80 + 5, at 10%. Find each total. Why are they different?

5. Discussion — 5 minutes

D. Does the worked example establish that every real tax is calculated after a discount? What information must a real question specify? Explain why P4 needs more than two percentage multipliers.

✓ 6. Closing check — 5 minutes

E. Start with 60. Decrease it by 10%, then increase the result by 10%. Find the intermediate amount, final amount and net percent change from 60.

Answer guide

- **O:** 0.10 × 90 = 9; the reference whole is 90.
- **G:** 50 × 1.20 = 60 centimetres; then 60 × 0.80 = 48 centimetres. The increase was 10 centimetres, but the decrease was 12. The net decrease is 2/50 = 0.04 = 4%, not zero.
- **P1:** Discounted price = 120 × 0.85 = 102. Tax = 102 × 0.10 = 10.20. Total = 112.20 currency units.
- **P2:** 200 × 1.10 = 220, then 220 × 1.10 = 242. Increase = 42; 42/200 = 0.21 = 21%. The second increase was 22, not 20.
- **P3:** 0.80 × original = 72, so original = 72/0.80 = 90 currency units. Check: 90 × 0.80 = 72. The proposed 72 × 1.20 = 86.40 adds 20% of 72, not the 18 removed from the original 90.

- **P4:** A: 80 × 1.10 + 5 = 93. B: (80 + 5) × 1.10 = 93.50 currency units. Policy B also taxes the fee, adding 0.50. Neither fictional policy is a claim about local law.
- **D:** “No: the problem must state the tax base and any fees. Real rules may differ. A flat fee is addition, so I cannot treat every operation as multiplication.” With the same two pure percentage factors and no intermediate rounding, their product commutes; this fact does not make all fee policies interchangeable.
- **E:** 60 × 0.90 = 54, then 54 × 1.10 = 59.40. The net decrease is 0.60/60 = 0.01 = 1%.

Vocabulary

- **Successive changes:** changes applied one after another, potentially with different reference wholes.
- **Reverse percent:** finding the original amount when the final amount and its percentage of the original are known.
- **Flat fee:** a fixed amount added, not a percentage of the amount bought.

Differentiation

- **Extra support:** provide two boxes headed “base for this step” and “new amount.” Complete the worked example with a bar before using a combined expression. Split successive changes and reverse percent into separate sessions if needed.
- **Extra challenge:** a final amount is 150 after a 25% increase. Find the original. **Guide:** 1.25x = 150, so x = 120; check 120 × 1.25 = 150.

⚠ Safety

Do not ask students to reveal family purchases, tax bills, debt or income. No real transactions are part of this lesson.

Teacher notes

Equal percentage increases and decreases generally do not cancel because the second base has changed. In reverse percent, undo the multiplier by division; do not merely switch “increase” and “decrease.” Require an explicitly stated base for every tax or fee example. Currency rounding policies vary in real life; the rule here is only an exercise assumption.

Previous lesson · Unit overview · Next lesson

Lesson 5: Choose the model, explain the result

Time: about 60 minutes (untested estimate; assessment may need a separate session)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Choose and explain proportional and percentage models, including a stated simple-interest model, and check results using units and reference wholes.

Materials

Paper, optional grid or tactile grid, ruler and calculator. No financial accounts, internet access or personal financial information are required.

Steps

✎ 1. Opening — 5 minutes

O. A stated proportional relationship includes (2, 9). Find k and write its equation.

✎ 2. Main activity: simple-interest worked example — 15 minutes

In a fictional arithmetic model, the principal, or starting amount, is 200 currency units. Simple interest is paid at 3% per year for two years, with no fees or payments in between. **Each year's interest is calculated on the original 200**, not on a growing balance.

One year's interest = 200 × 0.03 = 6 currency units. Two years' interest = 6 × 2 = 12. Final amount = 200 + 12 = 212 currency units.

Write I = Prt, where I is interest, P is principal, r is the annual rate written as a decimal, and t is time in years. Here I = 200 × 0.03 × 2 = 12. State that the formula describes this simple-interest assumption; it is not a universal account rule or an investment recommendation.

With P = 200 and r = 0.03 fixed, interest is I = 6t, proportional to time. But the **total amount** is A = 200 + 6t. It is not proportional to time: at zero years, the total is already 200, not zero. This connects the percent calculation to lesson 2's fixed-fee distinction.

3. Guided work — 5 minutes

G. Use simple interest with principal 150 currency units, annual rate 4%, and time three years. Find yearly interest, total interest and final amount. Explain which amount is the reference whole each year.

4. Individual synthesis and assessment — about 25 minutes

This is not a speed test; allow more time or another session. Give the prompts before the guide. Accept equations, labelled diagrams or oral explanations. Ask students to show the rate or percent base, not only a final number.

- **A1. Fractional rate.** At a constant price, 5/8 metre of ribbon costs 15/16 currency unit. Find currency units per metre and the cost of 2 metres.
- **A2. Table and equation.** Hours x are 0, 2, 5; costs y in currency units are 0, 7, 17.5. Does the table fit a proportional model? Give k, the equation and the meaning of (1, 3.5).
- **A3. Straight but not proportional.** The rule is y = 3 + 2x, where x is hours and y is currency units. Its table includes (0, 3), (1, 5), (2, 7). Explain why the rule is not proportional even though its graph is straight.
- **A4. Graph interpretation.** Volume x in litres and cost y in currency units follow y = 2x for continuous nonnegative volumes. Draw or describe a labelled graph including (0, 0), (1, 2) and (3, 6). Explain (3, 6) and how (0, 0) fits the model without dividing by zero.
- **A5. Change of base.** A count increases from 60 to 75, then later decreases from 75 to 60. Find the percent increase and percent decrease. Explain the difference.
- **A6. Successive percentages.** A fictional price of 150 currency units is discounted by 20%, then taxed at 5% of the discounted price. Find the final total, showing the tax base.
- **A7. Reverse percent.** An amount is 88 after a 10% increase. Find the original amount and check forward.
- **A8. Simple interest.** A fictional principal of 300 currency units receives 2% simple interest per year for four years, with no fees or intervening payments. Find the interest and final amount. Name the amount used as the percentage base each year.

5. Discussion — 5 minutes

D. Choose one assumption from today's models that would need checking in a real application. Explain what could change the answer if that assumption were false.

✓ 6. Closing check — 5 minutes

E. Explain why I = 5t is proportional to time t but A = 100 + 5t is not. Here t is years and both I and A are measured in currency units.

Worked answer guide

- **O:** k = 9/2 = 4.5; y = 4.5x. No units were specified in this abstract question.
- **G:** Yearly interest = 150 × 0.04 = 6 currency units; three years' interest = 18; final amount = 168. The original principal 150 is the reference whole every year.
- **A1:** (15/16) ÷ (5/8) = (15/16) × (8/5) = 3/2 currency units per metre. Two metres cost (3/2) × 2 = 3 currency units. Reverse units would answer a different question.
- **A2:** Yes: 7/2 = 17.5/5 = 3.5 currency units per hour; zero maps to zero. The fitted equation is y = 3.5x. (1, 3.5) means one hour costs 3.5 currency units under that model. The finite table fits; it does not alone prove unmeasured values.
- **A3:** At zero hours y = 3, not zero: the rule includes a fixed 3-currency-unit charge. Also 5/1 = 5 and 7/2 = 3.5 differ. The increase for each additional hour is constant, but the total cost per hour y/x is not.
- **A4:** Put volume in litres on the horizontal axis and cost in currency units on the vertical axis, with equally spaced numerical increments on each axis. The points lie on a straight line

through the origin; fractional inputs between them are permitted by the continuous-volume model. (3, 6) means 3 litres cost 6 currency units. At x = 0, y = 2 × 0 = 0; no 0/0 calculation is needed. The constant is 2 currency units per litre, read at (1, 2).
- **A5:** Increase = 15, based on 60: 15/60 = 0.25 = 25%. Decrease = 15, based on 75: 15/75 = 0.20 = 20%. The absolute changes match but their starting wholes differ.
- **A6:** Discounted price = 150 × 0.80 = 120. Tax base is 120, so tax = 120 × 0.05 = 6. Total = 126 currency units.
- **A7:** 1.10 × original = 88, so original = 88/1.10 = 80. Check: 80 × 1.10 = 88.
- **A8:** Yearly interest = 300 × 0.02 = 6 currency units. Four years' interest = 6 × 4 = 24. Final amount = 300 + 24 = 324. Every year's base is the original 300, not the balance after previous interest.
- **D:** Sample responses: “A fixed fee would change a pure cost-per-item model”; “Tax on the original rather than discounted price would change the tax”; “A rate observed during one interval might change later”; “Compound interest would use a growing balance rather than the original principal.” A specific assumption and consequence matter more than matching these words.
- **E:** I = 5t has constant I/t = 5 for positive t and gives I = 0 at t = 0. A = 100 + 5t starts at 100, so it is not of the form A = kt; its positive-time A/t is not constant. Its graph is nevertheless straight.

Feedback and next steps

Treat the competencies separately, not as a single speed or ability ranking:

- A1: if division is reversed, return to lesson 1's “one of what?” and units; if the setup is correct but arithmetic fails, support fraction division.
- A2–A4: distinguish a constant additional charge from a constant total ratio; revisit both the origin and a positive-input unit-rate point in lesson 2.
- A5: label the starting 100% whole separately for each direction, using lesson 3's bars.
- A6–A7: list each stage's base and check a reverse-percent answer forward, using lesson 4.
- A8: separate interest from final amount and simple from growing-balance rules.

After discussion, offer an analogous question rather than asking students to memorize this key. These prompts and feedback categories are not a validated assessment instrument; there is no established pass threshold.

Vocabulary

- **Principal:** the original amount used in the stated interest model.
- **Simple interest:** interest computed on the original principal for the stated time, rather than on accumulated interest.
- **Annual rate:** a rate per year; convert a percent to a decimal before using I = Prt.

Differentiation

- **Extra support:** provide one task per page, enlarged graphs or oral response options. Allow untimed work and a calculator after an expression is selected. Revisit the worked example before assessing simple interest separately.
- **Extra challenge, not required for assessment:** compare the worked example with annual compound growth at 3% for two years. **Guide:** 200 × 1.03 × 1.03 = 212.18,

compared with simple-interest total 212. The extra 0.18 is 3% of the first year's 6 interest. This illustrates a different assumption, not a full compound-interest course.

⚠ Safety

All money contexts are fictional. Do not request personal savings, borrowing, income or family circumstances. No purchases, account creation or financial recommendations are part of the activity.

Teacher notes

Separate “the interest is proportional to time” from “the total is proportional to time.” Keep rate periods and time units consistent: the displayed formula uses an annual rate with time in years. Classroom pacing and assessment use need local educator review. A second agent checking arithmetic does not make this a human-reviewed unit.

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