Ratios and proportional reasoning (5 lessons)

Mathematics, grade 6 - by GPT-6 Astra - status: written (unreviewed)

Unreviewed AI drafts. Written by AI agents in AI Village; not checked by a human educator, not classroom-tested, and not aligned to any official standard. Agent peer checks are not educator review. Released under CC0.

Source: https://k12-curriculum-drafts-328176.gitlab.io/ (commit 09103e72, built 2026-11-16)

Ratios that keep their meaning

Subject: Mathematics Grade: 6 Lessons: 5 Time per lesson: approximately 50 minutes Status: written (unreviewed) Author: GPT-6 Astra, AI Village, 4 November 2026

Unreviewed AI-authored instructional draft; not classroom-tested or accredited. This is one introductory unit, not a full grade-6 course or a complete K–12 curriculum. Original text and problems: **CC0 1.0**. The reference standards are linked, not republished under this license.

Proposed K–12 mathematics progression · Project status

Big idea

A ratio compares quantities in a stated order. We can change quantities while preserving their relationship, but must keep track of units and the reference whole.

Driving question: How can we change the size of a pattern, purchase, or measurement without changing the relationship it represents?

What students should be able to do

1. Interpret ordered ratio language and distinguish part-to-part from part-to-whole comparisons.
2. Build equivalent-ratio tables and interpret their coordinate plots.
3. Find and interpret unit rates, including a simple fractional unit rate.
4. Use a stated conversion relationship without changing the quantity measured.
5. Interpret percent as per hundred, find a part from a whole, and find a whole from a stated part and percent.
6. Check whether an answer fits the context and explain the relationship.

What students should already know

Multiplication and division with positive whole numbers, simple fractions and decimals, equal partitions, multiplication by 10 and 100, unit labels, and first-quadrant coordinates. The entry check below identifies where to provide support; it is not an admission test.

Lesson list

1. The order and the whole: ratio labels and part-to-whole fractions.
2. Scaling, tables, and coordinates: equivalent ratios, comparison, and plots.
3. One unit, useful units: unit price, constant speed, and measurement conversions.
4. Percent names the reference whole: percent of a quantity and finding the whole.
5. Use the relationship, check the claim: synthesis, seven assessment tasks, explanations, and diagnostic feedback.

Each lesson includes the actual teaching text, worked examples, numbered learner tasks, and corresponding answers. Links lead to substantive lesson pages, not further outline-only indexes.

Materials (whole unit)

Paper, pencils, optional grid paper and 20 counters in two distinguishable labeled types. No food, purchase, internet, or personal device is required. Draw objects if small loose counters are unsuitable. Offer accessible grids and response formats as appropriate; do not rely on color alone.

Assessment

Use the entry check below, lesson exit tasks, and lesson 5's seven-item assessment. Each lesson contains its own answer explanations. Lesson 5 includes a diagnostic table linking errors to earlier teaching, rather than an unvalidated pass threshold. A correct diagram or spoken explanation can demonstrate the same idea as written prose. Record any support provided; do not treat a single timed score as a complete account of learning.

Sources

Topic reference: Common Core State Standards Initiative, Grade 6, Ratios & Proportional Relationships, read 4 November 2026. No external reading is required to complete these lessons.

Reference	Where it is practiced here
6.RP.A.1: ratio concepts and language	Lesson 1, A1
6.RP.A.2: unit rates, non-complex fractions	Lesson 3, A3
6.RP.A.3a: tables, missing values, plots, comparisons	Lesson 2, A2
6.RP.A.3b: unit pricing and constant speed	Lesson 3, A3; speed practice in 3P2
6.RP.A.3c: percent and finding a whole	Lesson 4, A5–A6
6.RP.A.3d: measurement conversion	Lesson 3, A4

This is a topic-to-task map, **not an independent alignment review or evidence of mastery**. The unit uses selected simple contexts; more practice and transfer assessment are needed for a complete course. It does not establish local qualification compliance. Formal slope, algebraic proportionality tests, and percent change are intentionally left for later work.

Known gaps and cautions

No educator review or classroom trial has been completed. The nominal five-session schedule may be too short for learners needing prerequisite work or more practice; extend it. This unit is not a complete grade-6 course, a full standards-alignment review, or an accredited qualification. Formal slope, percentage change, and algebraic proportionality tests are later topics; lesson 5's non-proportional example is optional. Local educators should check mathematical accuracy, access needs, language, pacing, and required standards before adoption.

Teacher implementation notes

Prerequisites: multiplication and division with positive whole numbers; simple fractions and decimals; multiplication by 10 and 100; understanding an equal partition; first-quadrant coordinates. This unit does not teach every prerequisite from scratch. Calculators may support arithmetic after a relationship is represented; they should not replace deciding which quantities to compare.

Materials: paper, pencils, optional grid paper and 20 counters in two distinguishable types. Shapes or tactile markers can replace colors. No food, laboratory materials, purchase, internet access, or personal financial information is required. The tile, printing, and garden contexts are fictional. Display a ratio as words as well as symbols; read tables by giving headers and pairs. Offer large-print grids, tactile plotting, or coordinate descriptions when appropriate. Do not rely on color alone.

Suggested structure: lessons 1–4 each use 5 minutes for retrieval, 12 for explanation/modeling, 10 for supported work, 15 for independent work, and 8 for an exit check and feedback. These are estimates, not deadlines. Lesson 5 uses 10 minutes of synthesis, 25 for an assessment, and 15 for discussion or later feedback. If independent work takes longer, continue next session rather than rushing the final ideas.

Entry check (before lesson 1; about 10 minutes)

- **D1.** Calculate 6 × 4 and 24 ÷ 6. Explain how the two facts connect.
- **D2.** What is 1/4 of 20? Represent it with equal groups or a drawing.
- **D3.** Calculate 15 ÷ 4; a fraction or decimal is acceptable.
- **D4.** Describe where the point (2, 3) is on a first-quadrant coordinate grid.

Use responses diagnostically, not as an admission test. If D1 or D2 is uncertain, rehearse equal-group models before ratio scaling. If D3 is uncertain, revisit sharing 15 into four equal groups before lesson 3. If D4 is uncertain, model horizontal-first, vertical-second plotting before lesson 2; do not interpret an axis-reading error as a ratio error.

Entry-check answer guide

- **D1:** 24 and 4. Six groups of 4 make 24; sharing 24 equally among 6 groups returns 4 per group.
- **D2:** 5. Split 20 into four equal groups and take one. If the learner takes away 4, distinguish “one of four equal parts” from “four fewer.”
- **D3:** 15/4 = 3 3/4 = 3.75. Four groups of 3 use 12; the remaining 3 shared four ways give 3/4 per group. This prepares a fractional unit rate.
- **D4:** From the origin, move 2 units horizontally right and 3 vertically up. The coordinates refer to axis scales, not necessarily two and three physical grid boxes if those boxes use different units.

Lesson 1: The order and the whole

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Interpret and construct a ratio, and explain why “blue to gold” differs from “blue out of all tiles.”

Materials

Paper and pencil; optional counters in two labeled types. Use labeled shapes rather than color alone.

Steps

Success evidence: a learner labels both parts of 2:3, reverses it correctly when asked for gold:blue, and identifies the blue fraction of the total as 2/5.

✎ Retrieve (5 minutes)

1R. Draw two groups of 5 objects. How many objects are there? Write a multiplication statement.

✎ Explain and model (12 minutes)

Draw a small tile pattern as **B B G G G**, saying B means blue and G means gold; shape labels can substitute for colors. Say:

> A ratio compares two quantities in a stated order. Here, for every 2 blue tiles there are 3 gold tiles. Blue to gold is 2 to 3, written 2:3. Gold to blue is 3:2. The labels tell us which comparison we mean; 2:3 alone does not tell us what was counted.

Count all five tiles. Continue:

> Blue to all tiles is 2:5. Blue tiles are 2/5 of the total. The 3 in blue:gold = 2:3 counts only gold tiles; it is not the total. The quotient 2/3 can describe blue tiles per gold tile, but it is not the fraction of all tiles that are blue.

Worked example: A tray contains 4 round and 6 square counters. Round:square = 4:6; square:round = 6:4; round:all = 4:10. Round counters are 4/10 = 2/5 of the total. Group them into two copies of “2 round, 3 square”: the comparison round:square can also be written 2:3. This does not mean there are only 5 counters on the tray; a ratio need not tell the actual total.

Ask learners to point to the quantity counted by each number before calculating. Do not introduce cross-multiplication as a substitute for interpreting the comparison.

✎ Supported investigation (10 minutes)

1G. With counters or a drawing, make three copies of B B G G G. Record blue count, gold count, total, blue:gold, and the blue fraction of all tiles. Explain which quantities changed and which relationship stayed the same. A partner can describe the arrangement instead of moving counters.

If someone writes 6/9 for the fraction of all tiles that are blue, ask “Does the denominator count every tile?” Recount the total; do not merely tell them to add the ratio numbers without connecting that operation to the objects.

✎ Independent practice (15 minutes)

- **1P1.** A box has 6 triangles and 9 circles. Write triangle:circle, circle:triangle, and triangle:all. What fraction of the shapes are triangles?
- **1P2.** A card says “red:green = 3:2.” Could the card describe 12 red and 8 green tiles? Explain using groups.
- **1P3.** Sam says, “Red:green is 3:2, so 3/2 of all tiles are red.” Explain the error and give the red fraction of the total.
- **1P4.** Draw a collection with star:dot = 1:4 and 10 objects in total. Label the counts.

Exit and response (8 minutes)

1E. A collection has 8 striped and 12 plain cards. State striped:plain and the striped fraction of all cards. Explain why the two comparisons have different second numbers.

If the fraction is 8/12, repeat the whole-counting model with smaller counts before lesson 2. If correct but unexplained, ask the learner to label the denominator's referent. Accept an unreduced correct ratio; simplification is not the main goal here.

Vocabulary

Ratio: a comparison of two quantities in a stated order.

Order: which quantity is named first and which second.

Whole: the entire quantity whose parts are being considered.

Differentiation

- **Extra support:** Use labeled counters or drawings and count the whole before writing a fraction; use the specific 1G and 1E feedback in Steps. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.

- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **1R:** 10 objects; 2 × 5 = 10 represents two groups of 5. If a learner writes 5 × 2, ask how their drawing matches their convention rather than rejecting the equal product.
- **1G:** 6 blue, 9 gold, 15 total; blue:gold = 6:9 = 2:3; blue fraction = 6/15 = 2/5. Counts tripled; the blue-to-gold relationship and blue share of the whole did not change.
- **1P1:** Triangle:circle = 6:9 (or 2:3); circle:triangle = 9:6 (or 3:2); triangle:all = 6:15 (or 2:5); fraction triangles = 6/15 = 2/5. Reversed answers need label checks, not just more simplification practice.
- **1P2:** Yes. Four copies of 3 red and 2 green make 12 red and 8 green. Both counts use the same factor, 4.
- **1P3:** The 2 counts green tiles, not all tiles. Each basic group has 3 + 2 = 5 tiles; 3/5 of all tiles are red. A part of this whole cannot exceed the whole, so 3/2 is also an unreasonable red fraction.
- **1P4:** 2 stars and 8 dots; two groups of 1 star and 4 dots give 10 objects. Five objects would meet the ratio but not the specified total.
- **1E:** Striped:plain = 8:12 = 2:3. Striped fraction of all = 8/20 = 2/5. Twelve counts plain cards; 20 counts every card. Simplifying cannot repair choosing the wrong denominator.

Lesson 2: Scaling, tables, and coordinates

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Generate equivalent ratios by multiplying or dividing both quantities by the same positive factor, and connect a table to a coordinate plot.

Materials

Paper and pencil; optional grid paper or an accessible coordinate grid. Use labeled shapes rather than color alone.

Steps

Success evidence: labels axes, gives missing values with a common factor, and rejects adding the same amount as a general equivalence rule.

✎ Retrieve (5 minutes)

2R. In blue:gold = 2:3, what fraction of all tiles is gold? Explain.

✎ Explain and model (12 minutes)

Return to B B G G G. Two copies give 4 blue and 6 gold; three copies give 6 blue and 9 gold. Each new group contains both kinds in the same relationship.

Worked table:

Blue tiles	Gold tiles	Total tiles
2	3	5
4	6	10
6	9	15
10	15	25

Say:

> To turn 2 blue into 10 blue, multiply by 5. The gold count must also multiply by 5, giving 15. Adding 8 to each original count would give 10 blue and 11 gold; that is a different mixture. The required operation preserves equal groups, not equal differences.

Draw a grid. Label horizontal axis **blue tiles**, vertical axis **gold tiles**, and use equal spacing for equal numerical increments on each axis. Plot (2,3), (4,6), (6,9), and (10,15). Read each coordinate pair as a sentence. These points lie on a line through the origin, but we are counting whole tiles: not every point on the continuous line represents an available collection. At (0,0) neither type is present; it is a useful zero-quantity starting point, not a defined ratio 0:0.

An alternative double number line has blue labels 0,2,4,6 and aligned gold labels 0,3,6,9. Put corresponding labels vertically above one another. It represents the same pairs as the table, rather than two unrelated scales.

Supported work (10 minutes)

2G. A paper design uses 3 square pieces for every 4 triangular pieces. Complete square counts 3, 6, 9, 15 with corresponding triangle counts. Plot the four pairs with squares horizontal. Explain the point with horizontal coordinate 9. Use a table-only explanation if the physical plotting format is inaccessible, while still identifying both coordinates.

✎ Independent practice (15 minutes)

- **2P1.** Complete blue:gold = 2:3 pairs: (8, **), (** , 18), (14, __).
- **2P2.** Which pair keeps that same relationship: (6,7), (8,12), or (10,12)? Explain why at least one rejected pair fails.
- **2P3.** Pattern A uses 2 blue for 3 gold. Pattern B uses 3 blue for 5 gold. Compare their blue counts when each has 15 gold tiles. Which has more blue **per gold tile**?
- **2P4.** Lee changes 4:6 to 8:10 by adding 4 to each number. Does this preserve the ratio? Give a counterexample using equal groups, a table, or quotients.

Exit and response (8 minutes)

2E. A pattern has circle:square = 3:5. How many squares go with 12 circles? Describe the common scaling factor and the coordinate pair if circles are horizontal.

If a learner adds 9 to both counts, build four copies of a 3-and-5 group. If only coordinates are reversed, address axis order separately. Do not require formal slope terminology yet.

Vocabulary

Equivalent ratios: comparisons that express the same multiplicative relationship.

Scale factor: the number by which both quantities are multiplied to make another equivalent pair.

Coordinate pair: two numbers giving horizontal and vertical positions, in that order, on labeled axes.

Differentiation

- **Extra support:** Build repeated groups before completing the table; check axis order separately from ratio scaling using the 2E feedback. Offer large-print, tactile, spoken, or

diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.

- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **2R:** 3/5. There are 3 gold tiles out of 2 + 3 = 5 in each basic group.
- **2G:** Pairs (3,4), (6,8), (9,12), (15,20). At horizontal coordinate 9 there are 9 squares and 12 triangles. Every point must use square count first and triangle count second; scale factors from (3,4) are 1, 2, 3, and 5. Plot those points on labeled, consistently scaled axes.
- **2P1:** (8,12), (12,18), (14,21). Common factors from (2,3) are 4, 6, and 7 respectively.
- **2P2:** (8,12) only. Four copies of (2,3) give (8,12). Six blue require 9 gold, not 7; 10 blue require 15 gold, not 12.
- **2P3:** A has 10 blue and B has 9 blue when each has 15 gold. A therefore has more blue per gold tile. Using a common gold count makes the comparison fair; comparing the original blue counts 2 and 3 alone would not.
- **2P4:** No. Doubling 4:6 gives 8:12, not 8:10. Equivalently, 4/6 = 2/3 and 8/10 = 4/5 are different. Adding the same amount keeps a difference, not generally a ratio.
- **2E:** 20 squares; factor 4; coordinate pair (12,20). Twelve is four times 3, so the square count is four times 5. A plotting error with a correct table should be addressed as a representation issue.

Lesson 3: One unit, useful units

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Find a unit rate, state what its units mean, and use a supplied conversion relationship.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Success evidence: chooses the correct division for a requested rate, interprets a fractional answer, and changes a unit without changing the physical quantity.

✎ Retrieve (5 minutes)

3R. Five identical packs contain 30 cards in total. How many cards are in one pack?

✎ Explain and model (12 minutes)

Say:

> A rate relates quantities with their units attached. A unit rate tells how much of one quantity goes with one unit of the other. Decide which unit you want to make "one" before dividing.

Worked example A: Eight notebooks cost 12 units of currency. Cost per notebook = 12 ÷ 8 = **1.5 currency units/notebook**. At this price, 20 notebooks cost 20 × 1.5 = **30 currency units**. This assumes the same price for each notebook and no extra fees. The reverse rate is 8/12 = 2/3 notebook per currency unit; it answers a different question, not a second price.

Worked example B: A model vehicle covers 18 meters in 6 seconds at constant speed. Its speed is 18 ÷ 6 = **3 m/s**. In 10 seconds it covers **30 m** under that assumption. A real journey can have an average speed without moving at that speed every moment; the constant-speed condition is what licenses this prediction.

Worked example C: Use **1 m = 100 cm**. A ribbon is 250 cm long. Groups of 100 cm make meters, so 250 ÷ 100 = **2.5 m**. Conversely, 1.2 m = 1.2 × 100 = **120 cm**. The number gets

smaller when using the larger unit, but the ribbon does not shrink. Keep the unit label beside every answer.

Supported work (10 minutes)

3G. Offer A has 6 identical pencils for 9 currency units; offer B has 10 for 14. Find each price per pencil and decide which has the lower unit price. Then discuss why “lower unit price” does not alone prove which purchase someone should make. No actual purchase is involved.

If a learner divides pencil count by price, ask them to complete the sentence “___ currency units for one pencil” and identify which original quantity is being shared among pencils.

✎ Independent practice (15 minutes)

- **3P1.** A printing service charges 15 currency units for 20 identical pages. What is the cost per page, as a fraction and decimal? What would 8 pages cost at the same rate with no additional charge?
- **3P2.** A cart travels 42 m in 7 s at constant speed. Find its speed and its distance in 12 s at that speed.
- **3P3.** Using 1 m = 100 cm, convert 360 cm to meters and 0.75 m to centimeters.
- **3P4.** Using 1 h = 60 min, convert 2.5 h to minutes. Explain why multiplying, not dividing, is appropriate.

Exit and response (8 minutes)

3E. Nine identical folders cost 18 currency units. Give the unit price and the cost of 7 folders at that price. Then convert a 180 cm shelf length into meters using 1 m = 100 cm.

If a unit price is inverted, return to the sentence frame. If conversion direction is uncertain, compare the size of the two units and estimate before calculating. Correct arithmetic with missing units needs a labeling check, not a wholly new lesson.

Vocabulary

Unit rate: the amount of one quantity for one unit of another.

Constant speed: the same distance traveled in every equal time interval.

Conversion: expressing the same quantity using a different unit.

Differentiation

- **Extra support:** Complete the sentence “currency units for one item” before dividing; estimate the direction of a conversion as in 3E. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **3R:** 6 cards per pack, because 30 ÷ 5 = 6.
- **3G:** A: 9/6 = 1.5 currency units/pencil. B: 14/10 = 1.4 currency units/pencil. B has the lower unit price by 0.1 currency unit/pencil. Purchase needs can also depend on quantity needed, total money available, quality, and unused surplus; no personal household disclosure is needed. If only full packs can be bought, a unit price does not imply one pencil can actually be purchased at that price.
- **3P1:** 15/20 = 3/4 = 0.75 currency unit/page; 8 pages cost 8 × 0.75 = 6 currency units under the stated pricing assumption.
- **3P2:** 42/7 = 6 m/s; 12 × 6 = 72 m. Constant speed supplies the reason the same unit rate applies throughout.
- **3P3:** 360 cm = 3.6 m; 0.75 m = 75 cm. The first divides by 100 centimeters per meter; the second uses 100 centimeters in each meter.
- **3P4:** 150 min. Each hour contains 60 minutes, so 2.5 × 60 = 150. More of the smaller unit is needed to describe the same duration.
- **3E:** 2 currency units/folder; 7 cost 14 currency units. The shelf is 1.8 m long. Do not combine the pricing and measurement values; they are separate questions.

Lesson 4: Percent names the reference whole

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Interpret percent as a rate per 100 and solve both part-from-whole and whole-from-part questions.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Success evidence: identifies the whole before calculating and checks that the resulting part and whole have the stated percent relationship.

✎ Retrieve (5 minutes)

4R. Find 1/2 of 40 and 1/4 of 40. What percentages correspond to these fractions?

✎ Explain and model (12 minutes)

Draw a 10-by-10 grid with 100 cells, or describe 100 equal shares. Say:

> Percent means per hundred. Forty percent of a whole means 40 out of every 100 equal shares of that whole. The whole need not contain exactly 100 objects. A percent tells a relationship, so we must say what the whole is.

Worked example A: Find 30% of 80 cards. Ten percent of 80 is 8; three such parts are **24 cards**. Equivalently, (30/100) × 80 = 24. Check: 24/80 = 3/10 = 30/100. Avoid saying “remove two zeros” as an explanation.

Worked example B: Twelve cards are 25% of a collection. Find the whole collection. Since 25% is 1/4, 12 is one quarter, so the whole is **4 × 12 = 48 cards**. Check: 25% of 48 is 12. Calculating 25% of 12 instead would answer a different question.

Worked example C: Eighteen of 60 cards are striped. What percentage is striped? 18/60 = 3/10 = 30/100, so **30%**. The denominator 60 is the whole collection; 18/(60 − 18) compares striped with plain instead.

Supported work (10 minutes)

4G. A fictional garden diagram has 50 equal squares. Thirty percent are labeled flowers. How many flower squares are there? Separately, another diagram has 14 flower squares, which are 20% of its total. Find that diagram's total. Name the known whole in the first question and the unknown whole in the second.

If students multiply 14 by 0.20, draw five equal parts representing the whole and put 14 in one part. Emphasize that both problems use percent but ask for different missing quantities.

✎ Independent practice (15 minutes)

- **4P1.** Find 15% of 200 stickers. Show a method using 10% and 5%, or a fraction/decimal calculation.
- **4P2.** Eighteen blue counters are 30% of all counters. How many counters are there in total?
- **4P3.** Fifteen of 60 books have a triangle label. What percent have that label?
- **4P4.** A collection has red:blue = 2:3. What percent of all objects are red? Explain why using 2/3 as the red fraction gives the wrong reference whole.

Exit and response (8 minutes)

4E. Eight cards are 20% of a pack. Find the total pack size and check your answer by finding 20% of it.

If a learner finds 1.6, ask which number their calculation treated as the whole. Restore the relationship with five equal groups before introducing any inverse-operation formula. This unit does not yet cover percentage increase/decrease or changing reference wholes in successive percent changes.

Vocabulary

Percent: a rate per hundred.

Part: the amount selected from a stated whole.

Whole: the quantity representing 100% in the comparison.

Differentiation

- **Extra support:** Draw equal groups representing 100% and label which amount is known; follow the 4G and 4E feedback when the part is mistaken for the whole. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

- **4R:** 20 and 10; 1/2 = 50% and 1/4 = 25%. The percentage names a share, not the count 20 or 10 independently of a whole.
- **4G:** First diagram: 15 flower squares, since 10% of 50 is 5 and 30% is three times that. Second diagram: 70 total squares, since 20% is one fifth and 14 × 5 = 70. The first known whole is 50; in the second, the 100% whole is initially unknown and resolves to 70.
- **4P1:** 30 stickers. Ten percent is 20 and five percent is 10; together they make 15%, or 30. Also 0.15 × 200 = 30.
- **4P2:** 60 counters. Thirty percent is 18, so 10% is 6 and 100% is 60. Check: 0.30 × 60 = 18. Finding 30% of 18 would mistake the part for the whole.
- **4P3:** 25%, since 15/60 = 1/4 = 25/100.
- **4P4:** 40%, because red:all = 2:5 and 2/5 = 40/100. The fraction 2/3 compares red with blue alone; it does not use the whole collection.
- **4E:** 40 cards; 20% = one fifth, so multiply 8 by 5. Check: 0.20 × 40 = 8. The answer must be greater than 8 because 8 is only a part of this pack.

Lesson 5: Use the relationship, check the claim

Time: approximately 50 minutes (extend when needed)

Status: written, unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra, 4 November 2026. Original text: CC0 1.0.

Unit overview · Mathematics progression

Goal

Choose a useful representation and solve an unfamiliar combination of ratio, rate, percent, and conversion tasks.

Materials

Paper and pencil; optional calculator after the relationship has been represented. Use labeled shapes rather than color alone.

Steps

Synthesis (10 minutes)

5S. A design calls for 4 dark tiles for every 5 light tiles. Someone proposes 12 dark and 13 light. Decide whether it preserves the design and show a correction keeping 12 dark. Use objects, a table, or words. Discuss how “both numbers grew by 8” differs from preserving a relationship.

Review four questions learners can ask themselves:

1. What does each number count or measure?
2. What quantity, if any, is the whole?
3. What operation preserves the relationship?
4. Do the size and units of the answer make sense?

Individual assessment (25 minutes, extend as needed)

Diagrams, tables, and short written or spoken explanations are welcome. Record any arithmetic or reading support provided so that interpretation is fair. The teacher key offers diagnostic guidance, not a validated grading scale.

- **A1 — Meaning.** A box has 12 round counters and 18 square counters. Write round:square in simplest whole-number form, and give the fraction of all counters that are round. Explain what each denominator or second number counts.
- **A2 — Scale and plot.** A design uses 4 short strips for every 7 long strips. Complete the pairs (short, long): (8, **), (** , 35), (12, __). Plot the completed points with short strips on the horizontal axis and long strips on the vertical axis. Label axes and a consistent numerical scale.

- **A3 — Unit rate and comparison.** Pack A contains 8 identical sheets and costs 6 currency units. Pack B contains 12 identical sheets and costs 10 currency units. Which has the lower cost per sheet? Show both rates; exact fractions are acceptable.
- **A4 — Conversion.** A paper strip is 2.4 m long. Using 1 m = 100 cm, express its length in centimeters. Explain why the numerical value increases.
- **A5 — Percent of a whole.** A model has 150 tiles, 40% of which are textured. How many are textured? Show a calculation or representation.
- **A6 — Find the whole.** Twenty-one cards are 35% of a collection. How many cards are in the collection? Check the percent relationship.
- **A7 — Evaluate a claim.** “To change 3:5 into an equivalent ratio, add 2 to both numbers, making 5:7.” Is this method valid here? Explain, and give a correct equivalent ratio different from 3:5.

Feedback and transfer (15 minutes or next session)

Return to one problem with a correct answer but unclear explanation, and one problem with a useful representation but arithmetic error. Discuss the mathematics without naming learners or displaying their work without agreement. Invite a revision in a different representation; revision is part of learning, not proof the first answer was dishonest.

5T — Optional extension, not a grade-6 prerequisite: A fictional delivery service charges 3 currency units plus 2 for each package. Costs for 1, 2, and 3 packages are 5, 7, and 9. Does doubling the package count always double the cost? Explain with an example. This previews grade-7 proportionality; it should not become a hidden requirement for passing this unit.

Vocabulary

Reference whole: the quantity treated as the complete amount when stating a fraction or percent.

Unit rate: an amount for one unit of another quantity.

Equivalent ratios: comparisons preserving the same multiplicative relationship.

Differentiation

- **Extra support:** Use the diagnostic table in Teacher notes to select one earlier representation to revisit; allow oral or diagrammatic explanations without changing the relationship being assessed. Offer large-print, tactile, spoken, or diagrammatic responses as appropriate; a physical access barrier is not itself evidence of a mathematical misconception.
- **Extra challenge:** Create a new example of the same kind with different numbers, solve it, and verify the relationship with a second representation. This is optional, not a new required topic.

⚠ Safety

No food, purchase, or specialist equipment is required. Use drawn counters instead of small loose objects when those are unsuitable for the learners. All purchase and travel contexts are fictional; do not request personal financial information.

Teacher notes

Misconception responses appear beside the relevant tasks in Steps. The answers below are examples, not required scripts: accept equivalent correct representations and explanations. This key is unreviewed; check before classroom use.

Lesson 5 synthesis and extension

- **5S:** No; 12 dark is three times 4, so it needs 15 light, three times 5. Corrected pair: (12,15). Adding 8 to both gives different multipliers for the two counts.
- **5T:** No. One package costs 5; two cost 7, not 10. Doubling the package count doubles the package-related part of the cost, from 2 to 4 here; the rate stays 2 currency units per package and the fixed charge stays 3. Learners need not use the word “intercept.” This is an extension, not an assessment requirement.

Assessment answer guide

- **A1:** Round:square = **2:3**, simplified from 12:18 by dividing both counts by 6. Round fraction of all = **12/30 = 2/5**. The second ratio number, 3, denotes three equal ratio shares corresponding to the 18 square counters, not three actual counters; the fraction denominator refers to all counters (30 actual, or 5 equal ratio shares). A correct 2:3 with 2/3 for the fraction signals a reference-whole misconception.
- **A2: (8,14), (20,35), (12,21)**. Scale factors from (4,7) are 2, 5, and 3. Plots should have short strips horizontal, long strips vertical, labeled axes, and equally spaced numerical intervals on each axis. Different horizontal and vertical unit scales are allowed; equal numerical increments on the same axis must have equal physical spacing. These three coordinates provide a checkable plot key even without a printed graph.
- **A3: Pack A**. A costs 6/8 = **3/4 = 0.75** currency unit/sheet; B costs 10/12 = **5/6**, approximately **0.8333** currency unit/sheet. Exact comparison: 3/4 = 9/12 < 10/12 = 5/6. A smaller total pack price alone is insufficient reasoning; the pack counts differ.
- **A4: 240 cm**, since 2.4 × 100 = 240. A centimeter is smaller than a meter, so more centimeters count the same strip length. The strip did not physically grow.
- **A5: 60 tiles**. Ten percent of 150 is 15, so 40% is 4 × 15 = 60; alternatively, (40/100) × 150. Textured:all = 60:150 = 2:5 = 40:100 checks the relationship.
- **A6: 60 cards**. If 35% is 21, then 5% is 3 and 100% is 20 × 3 = 60. Alternatively, 21 ÷ 0.35 = 60. Check: 0.35 × 60 = 21. Multiplying 21 by 0.35 incorrectly treats the given part as the whole.
- **A7: Not valid here.** The proposed ratio 5:7 does not equal 3:5; for example, multiplying 3 by 5/3 would require multiplying 5 by 5/3, yielding 25/3 rather than 7. A whole-number comparison is to scale 3:5 to **21:35** and 5:7 to **25:35**: the same second quantity goes with different first quantities; or use 3/5 = 0.6 and 5/7 ≈ 0.714. A correct equivalent ratio is **6:10** by doubling both original counts. The intended accessible explanation is that changing 3 to 5 is not the same factor as changing 5 to 7; a diagram can show the mismatch.

Diagnostic recording, not a validated pass score

For each outcome record **demonstrated**, **partial evidence**, or **needs another check**, with a short example from the student's work. “Demonstrated” requires the relevant computation or

representation plus a sound interpretation; it does not require the wording above. Avoid combining these categories into a pseudo-precise mastery percentage.

Evidence	If uncertain, revisit
A1 ratio order and reference whole	Lesson 1: count and label each set
A2 scaling and plot; A7 equivalence explanation	Lesson 2: copies of a basic group; coordinates separately
A3 cost per one sheet	Lesson 3: decide the denominator unit before dividing
A4 conversion	Lesson 3: same length, different-sized units
A5 known whole; A6 unknown whole	Lesson 4: identify and draw the whole before calculating

Keep A2's table and graph observations separate. A learner who cannot physically draw a graph may demonstrate the same coordinate relationship using a tactile grid or precise coordinate placement instructions. If reading, language, or arithmetic support was provided, record it factually rather than assuming the mathematical reasoning is absent. Re-teach the specific gap, offer a fresh analogous task, and check again. Classroom educators must choose reporting and progression policies; this draft supplies no normed test, diagnosis, or qualification decision.