Proportional relationships: what stays constant? (5 lessons)

Mathematics, grade 7 - by GPT-6 Astra - status: written (unreviewed)

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Proportional relationships: what stays constant?

Subject: Mathematics Grade: 7 Lessons: 5 Time per lesson: about 60 minutes

Status: written (unreviewed AI draft; not human-educator-reviewed or classroom-tested)

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Big idea

In a proportional relationship, multiplying one quantity by a factor multiplies the other by the same factor. Its equation is y = kx: the rate y/x stays constant for positive x. A percentage calculation also needs a clearly identified reference whole, which can change from one step to the next.

What students should be able to do

- Compute a unit rate from fractional quantities, label its units, and distinguish it from the inverse rate.
- Distinguish proportional from nonproportional relationships in tables, verbal rules and graphs.
- Write y = kx and explain the constant k and points (0, 0) and (1, k), using the quantities and units on the axes.
- Calculate percent increase or decrease and successive percentages with the correct reference whole at each step.
- Find an original amount from a stated final percentage and explain a simple-interest model.

What students should already know

Students need whole-number and fractional unit rates, percent of a whole, fraction multiplication and division, decimal operations, and nonnegative coordinate pairs. The grade 6 ratios unit supplies the ratio and percent foundation; this is not a replacement for instruction in fraction division.

Use these three diagnostic questions without a time limit. Ask for a representation or explanation as well as a result:

1. Three identical notebooks cost 6 currency units, with no fee. What is the cost per notebook?
2. Find 25% of 80.
3. How many halves are in three quarters? Calculate (3/4) ÷ (1/2).

Diagnostic guide: (1) 6 ÷ 3 = 2 currency units per notebook. (2) One quarter of 80 is 20. (3) Three quarters contains one and a half halves, so the quotient is 3/2. A strip divided into quarters makes the comparison visible. These questions are not an admission gate: revisit equal groups, a 100% bar, or fraction strips where needed before combining the ideas.

Lesson list

1. A rate when both quantities are fractions: compute and interpret rates, including inverse units.
2. A straight line is not enough: connect rules, tables, graphs and the origin.
3. Percent change needs a starting whole: distinguish change from final amount and identify the denominator.
4. Two percentages, two reference wholes: successive changes, reverse percent and fees.
5. Choose the model, explain the result: simple interest, synthesis and an eight-task assessment.

Materials (whole unit)

Paper, pencil, ruler, and optional grid paper or tactile grid; fraction strips can be drawn rather than purchased. Offer large-print pages and verbal descriptions. A calculator is optional after a student has written or described the intended calculation. No internet, purchases, food preparation or personal financial information is required.

All currency amounts and rates are fictional arithmetic examples. Substitute a locally familiar currency label if helpful, without changing the stated mathematical rule. These are not actual tax rules or financial advice. Counts of indivisible packs use whole-number inputs; lengths, times and volumes may use fractional inputs.

Assessment

Use lesson 5's A1–A8 tasks and worked guide. Accept oral explanations, labelled drawings or equations; do not score speed or handwriting. Look separately for rate direction and units, proportional-model recognition, graph interpretation, and correct percent bases. Use errors to select a worked example for revisiting, then offer a similar new question. There is no validated pass threshold or claim that eight tasks measure all grade 7 mathematics.

Sources

The Common Core grade 7 Ratios and Proportional Relationships reference was consulted on November 4, 2026. 7.RP.A.1 informed fractional unit rates; 7.RP.A.2a–d informed tables, graphs, equations and point interpretation; 7.RP.A.3 informed multi-step ratio and percent problems. This is a scope cross-reference, not accreditation, a complete alignment audit, or a full grade 7 course. Examples and explanations here are original CC0 material; the linked reference is not relicensed by this repository.

See also the proposed mathematics progression.

Known gaps and cautions

Pacing is an untested estimate. Split lessons, especially lessons 4 and 5, if explanation or practice needs more time. A qualified local educator should check language, access needs, prerequisites, curriculum requirements and examples before classroom use. An agent peer check is not human educator review.

The unit uses nonnegative quantities and does not develop general functions on negative inputs. It is not a finance course. Compound interest appears only as an optional contrast; percent error, commission and other contexts are not exhaustively covered. Data agreeing with a model at

several points does not prove that it holds at unobserved values. Explicit rules in the exercises are assumptions, not empirical claims about equipment, motion or prices.

Lesson 1: A rate when both quantities are fractions

Time: about 60 minutes (untested estimate; split if needed)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Compute a unit rate from fractional quantities and explain why reversing the division changes both the rate and its units.

Materials

Paper, pencil, optional fraction strips and calculator. Ribbon is a fictional measured quantity: no cutting or purchases are needed.

Steps

✎ 1. Opening — 5 minutes

O. In a fictional shop, 3 metres of ribbon cost 6 currency units, with a constant price per metre and no fee. Find the price per metre. Ask students to name the quantity that should become **one**, rather than starting with a memorized division rule.

✎ 2. Main activity: worked model — 15 minutes

Display: “Three quarters of a metre cost nine eighths of a currency unit, at a constant price per metre.” Write the question in words: **How much currency for one metre?**

Draw a one-metre strip divided into four quarters. Label three quarters together 9/8 currency units. One quarter costs (9/8) ÷ 3 = 3/8. Four quarters cost 4 × 3/8 = 12/8 = 3/2 currency units.

Length (metres)	Cost (currency units)
0	0
1/4	3/8
3/4	9/8
1	3/2

Connect the strip to division: (9/8) ÷ (3/4) = (9/8) × (4/3) = 3/2 **currency units per metre**. Multiplying both the length and the cost by 4/3 turns 3/4 metre into one metre. The fraction calculation and the strip describe the same scaling.

Write cost = (3/2) × length, when length is measured in metres. For continuous ribbon sold by length, fractional metre inputs are allowed in this model.

Now reverse the question: “How many metres for one currency unit?” The calculation is (3/4) ÷ (9/8) = 2/3 **metre per currency unit**. Both rates are correct, but they answer different questions. Do not drop the units when comparing them.

3. Guided work — 10 minutes

G. A constant-motion model travels 5/6 kilometre in 1/3 hour. Find kilometres per hour and the distance in 1/2 hour. Have students first scale the time to one hour, then use the rate for half an hour. A single journey's average rate does not by itself establish that the speed stays constant; here constancy is part of the stated model.

✎ 4. Independent practice — 20 minutes

For each task, write the division with units or describe the scaling. A calculator may check the arithmetic after the model is chosen.

- **P1.** A constant-price ribbon model gives 2/3 metre for 5/6 currency unit. Find currency units per metre and the cost of 2 metres.
- **P2.** In a constant-flow model, 3/5 litre fills in 2/5 minute. Find litres per minute and the time needed for 2 litres.
- **P3.** A length of 3/8 metre costs 3/4 currency unit. Find both currency units per metre and metres per currency unit.
- **P4.** Someone divides 3/4 metre by 9/8 currency units and reports “2/3 currency unit per metre.” Explain the mistake and give the price per metre.

5. Discussion — 5 minutes

D. Why can two different numbers both be correct unit rates for the same purchase? Ask for the question each answers, not just the phrase “flip it.”

✓ 6. Closing check — 5 minutes

E. A constant-motion model travels 7/10 kilometre in 1/5 hour. Find kilometres per hour and the distance in 2/5 hour.

Answer guide

- **O:** 6 ÷ 3 = 2 currency units per metre; multiplying 2 by 3 metres gives the original cost 6.
- **G:** (5/6) ÷ (1/3) = (5/6) × 3 = 5/2 kilometres per hour. Half an hour gives (5/2) × (1/2) = 5/4 kilometres. Alternatively, half an hour is 3/2 times the given time, so multiply the given distance by 3/2 too.
- **P1:** (5/6) ÷ (2/3) = 5/4 currency units per metre. Two metres cost (5/4) × 2 = 5/2 currency units.
- **P2:** (3/5) ÷ (2/5) = 3/2 litres per minute. Time = 2 ÷ (3/2) = 4/3 minutes. Check: (3/2) × (4/3) = 2 litres.
- **P3:** (3/4) ÷ (3/8) = 2 currency units per metre; (3/8) ÷ (3/4) = 1/2 metre per currency unit. The reciprocal has different units.
- **P4:** The division produces 2/3 **metre per currency unit**, not currency per metre. For the requested price, (9/8) ÷ (3/4) = 3/2 currency units per metre.

- **D:** “One rate tells me the cost of one metre; the other tells me the length bought for one currency unit. I must say which one I want.”
- **E:** (7/10) ÷ (1/5) = 7/2 kilometres per hour. In 2/5 hour the distance is (7/2) × (2/5) = 7/5 kilometres. This is twice the original distance because the time doubled.

Vocabulary

- **Unit rate:** amount of one quantity for one unit of another, with units specified.
- **Inverse rate:** the rate for the reversed question; for a nonzero rate its numerical value is the reciprocal when the units are reversed consistently.
- **Constant rate:** a rate that does not change within the stated model.

Differentiation

- **Extra support:** use the quarter-metre strip before symbolic fraction division. Read “currency divided by metres” aloud. Allow an oral answer with a labelled strip.
- **Extra challenge:** express the worked ribbon cost as y = kx. **Guide:** x is metres, y is currency units, k = 3/2 currency units per metre. Fractional nonnegative x values are meaningful for this continuous-length model.

⚠ Safety

Use paper models. Do not enact speed or liquid-flow experiments. No purchases or disclosure of personal spending are needed.

Teacher notes

A correct number with reversed units is not yet a correct interpretation. Ask “one of what?” before selecting the division. Fraction strips explain the scaling; a rule for dividing fractions should not replace that explanation. Keep the constant-rate assumption explicit when moving from an observed pair to another time or length.

Unit overview · Next lesson

Lesson 2: A straight line is not enough

Time: about 60 minutes (untested estimate)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Recognize a proportional relationship and connect its constant rate, equation and graph, including the meaning of the origin and the unit-rate point.

Materials

Paper, ruler, optional grid or tactile grid and calculator. Graphs may be described orally with labelled coordinates.

Steps

✎ 1. Opening — 5 minutes

O. Four identical cards cost 6 currency units, with a constant price per card and no fee. What does one card cost? What do eight cost? Name the assumption that permits the second calculation.

✎ 2. Main activity: worked comparison — 15 minutes

Two fictional equipment-use models relate hours x to cost y. Model A charges 3 currency units per hour with no fee: y = 3x. Model B charges a fixed access fee of 2 currency units plus 3 per hour: y = 2 + 3x. The access fee applies even at zero usage once access is purchased.

Hours x	A: cost y	B: cost y
0	0	2
1	3	5
2	6	8
4	12	14

For positive x, A has y/x = 3 at every listed point. B has ratios 5/1 = 5, 8/2 = 4, and 14/4 = 3.5. Its cost per hour of use is not constant even though the charge **for each additional hour** is constant.

Draw horizontal hours from 0 to 4 and vertical currency units from 0 to 14. Use equal spacing for equal numerical increments on each axis; the two axes need not use the same physical scale. Label the axes and plot both sets. Join each model's points with a straight line over nonnegative hours, since this example allows fractional hours and explicitly gives a linear rule. Only A's line passes through the origin.

Explain A's points: (0, 0) means no hours and no cost; (1, 3) means one hour costs 3 currency units, so k = 3 currency units per hour; (2, 6) means two hours cost 6. We do **not** calculate 0/0 at the origin. The rate is determined at positive x, and y = kx gives zero output at zero input.

A straight line alone is insufficient: B is straight. Passing through the origin alone is also insufficient: a curved or irregular relation can include (0, 0). A proportional graph has both properties, over its allowed domain. For discrete counts, the allowable points lie on such a line, but intermediate points may not be valid quantities.

3. Guided work — 10 minutes

G. Ribbon cost y in currency units is y = 2.5x, where x is length in metres. Plot or describe (0, 0), (1, 2.5), (4, 10) and find the point for 3 metres. Explain k and the unit-rate point.

✎ 4. Independent practice — 20 minutes

- **P1.** A table has x values 0, 2, 4, 6 metres and y values 0, 5, 10, 15 currency units. Does it fit a proportional model? State k and its equation. **Assuming that model continues**, find y when x = 10.
- **P2.** A table has x values 0, 1, 2, 3 and y values 0, 2, 5, 9. Does it fit a proportional model? Explain why the origin does not settle the question.
- **P3.** For length x in metres and cost y in currency units, compare y = 4x with y = 4 + x. Which is proportional? For the proportional model, explain (1, 4).
- **P4.** A stated proportional relationship includes (2, 7), with x in hours and y in currency units. Find k, the equation, the unit-rate point and the point for zero hours.

5. Discussion — 5 minutes

D. Does agreement with a proportional model at three measured positive inputs prove that all future measurements will fit it? Distinguish a pattern in data from an explicitly assumed rule.

✓ 6. Closing check — 5 minutes

E. Identical whole packs have x values 0, 3, 6 and costs y of 0, 12, 24 currency units, with a constant price per pack and no fee. Find k. Explain why (1, 4) is a valid purchase but (1/2, 2) is not allowed when only whole packs are sold, even though both lie on y = 4x.

Answer guide

- **O:** 6/4 = 1.5 currency units per card; eight cost 8 × 1.5 = 12. The price per card remains constant and no fixed fee is added.
- **G:** k = 2.5 currency units per metre. (1, 2.5) represents one metre costing 2.5 currency units. At 3 metres, y = 2.5 × 3 = 7.5, so the point is (3, 7.5). The stated rule gives a straight line through (0, 0).
- **P1:** Yes: 5/2 = 10/4 = 15/6 = 2.5 currency units per metre, and zero maps to zero. The fitted model is y = 2.5x; under the continuation assumption, y = 25 currency units at 10 metres.

- **P2:** No: the positive-input ratios are 2, 2.5 and 3, not one constant. Having (0, 0) does not fix this mismatch.
- **P3:** y = 4x is proportional, with k = 4 currency units per metre. (1, 4) means one metre costs 4 currency units. y = 4 + x is not proportional: zero length has cost 4, and the positive-input ratios differ.
- **P4:** k = 7/2 = 3.5 currency units per hour; y = 3.5x; unit-rate point (1, 3.5); zero-hours point (0, 0). No division by zero is used.
- **D:** “The measurements fit the model so far, but an unmeasured value might not. If the exercise gives the rule for the whole domain, I can calculate from that assumption.”
- **E:** k = 12/3 = 24/6 = 4 currency units per pack. One whole pack costs 4. The half-pack point is on the mathematical line but outside the allowed whole-pack inputs in this situation.

Vocabulary

- **Constant of proportionality k:** the fixed multiplier in y = kx; y/x for positive x in these examples.
- **Origin:** the point (0, 0).
- **Domain:** the allowed input values in a model.
- **Linear but nonproportional:** a straight-line relationship with a nonzero intercept, such as y = 2 + 3x.

Differentiation

- **Extra support:** say each coordinate as “horizontal quantity, then vertical quantity.” Use a tactile grid, enlarged ticks or oral descriptions. Compare A and B at x = 0 before computing rates.
- **Extra challenge:** reverse A's axes. **Guide:** hours = (1/3) × cost, so the reversed constant is 1/3 hour per currency unit, the reciprocal of 3 currency units per hour.

⚠ Safety

This is a paper activity; no equipment hire or other purchase is needed.

Teacher notes

Equal increases in y for equal increases in x establish a straight-line pattern in a table, not necessarily proportionality. Avoid calling y = 2 + 3x proportional because its “rate” is 3: its slope is 3, but its total y/x is not constant. A finite table is evidence of fit, not proof about unobserved data. For discrete contexts, draw or describe allowable points separately from the containing line.

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Lesson 3: Percent change needs a starting whole

Time: about 60 minutes (untested estimate)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Calculate percent increase or decrease using the original amount as the reference whole, and distinguish the change from the final amount.

Materials

Paper, pencil, optional calculator and drawn or tactile percentage bar.

Steps

✎ 1. Opening — 5 minutes

O. Find 30% of 50. Name the whole and say whether your answer is the part or the whole.

✎ 2. Main activity: worked comparison — 15 minutes

Display: “A collection grows from 80 counters to 100 counters.” The increase is 100 − 80 = 20 counters. Percent increase compares that change to the **starting** 80:

percent increase = (20/80) × 100% = 25%.

Draw a bar for 80 counters, labelled 100%. Divide it into four equal parts: each represents 20 counters, or 25% of the original. Add one more such part. The final 100 counters represent 125% of the original 80, not 25% of it.

Connect the representations:

- Increase: 0.25 × 80 = 20 counters.
- Final amount: 80 + 20 = 100 counters.
- Equivalent multiplier: 1.25 × 80 = 100 counters.

Now reverse the journey: “A collection shrinks from 100 counters to 80.” The decrease is still 20 counters, but the starting whole is now 100. Percent decrease = (20/100) × 100% = 20%. Draw a fresh bar labelled 100 counters = 100%; do not reuse the old 100% label on 80.

For a percent change written as a decimal r, an increase gives final amount = original × (1 + r); a decrease gives final amount = original × (1 − r). These percent-change calculations require a positive original amount. Here decreases have $0 \le r \le 1$ so the final amount is

nonnegative. A 100% decrease leaves zero; an increase can exceed 100%. Multipliers describe the final amount, not just the change.

3. Guided work — 10 minutes

G. A length changes from 60 centimetres to 45 centimetres. Find the decrease in centimetres, the percent decrease, and the final length as a percentage of the original. Ask students to label 100% before calculating.

✎ 4. Independent practice — 20 minutes

For each answer, name the original whole and include a checking calculation.

- **P1.** A count grows from 50 to 65. Find the percent increase.
- **P2.** A length shrinks from 120 centimetres to 90 centimetres. Find the percent decrease.
- **P3.** Increase 30 counters by 40%. Find the increase and the final count; explain why the two answers differ.
- **P4.** Someone says, “80 to 100 is a 20% increase because 20/100 = 0.20.” Identify the wrong reference whole. What is the correct percent increase, and what is the percent decrease for 100 to 80?

5. Discussion — 5 minutes

D. Why do the forward and backward changes between 80 and 100 have different percentages even though both involve 20 counters?

✓ 6. Closing check — 5 minutes

E. A count grows from 200 to 230. Find the percent increase. Complete: “230 is ___% of 200.”

Answer guide

- **O:** 0.30 × 50 = 15. The whole is 50; 15 is the 30% part. It is not a new total unless a problem specifies how that part is used.
- **G:** Decrease = 60 − 45 = 15 centimetres. Percent decrease = (15/60) × 100% = 25%. Final length is 75% of the original: 0.75 × 60 = 45 centimetres.
- **P1:** Increase = 65 − 50 = 15; (15/50) × 100% = 30%. Check: 1.30 × 50 = 65.
- **P2:** Decrease = 120 − 90 = 30 centimetres; (30/120) × 100% = 25%. Check: 0.75 × 120 = 90 centimetres.
- **P3:** Increase = 0.40 × 30 = 12 counters. Final count = 30 + 12 = 42 counters, or 1.40 × 30. The increase alone omits the original 30 counters.
- **P4:** The increase starts at 80, not 100: 20/80 = 0.25, so the increase is 25%. The reverse starts at 100: 20/100 = 0.20, so the decrease is 20%.
- **D:** “The numerator, 20 counters, is the same, but the denominator is the starting whole. Twenty is a larger fraction of 80 than of 100.”
- **E:** Increase = 30; (30/200) × 100% = 15%. The final 230 is 115% of 200 because 1.15 × 200 = 230.

Vocabulary

- **Reference whole:** the amount that represents 100% in a calculation.
- **Percent increase/decrease:** the increase/decrease divided by the original positive amount, expressed as a percentage.
- **Multiplier:** the factor applied to the original amount to obtain the final amount.

Differentiation

- **Extra support:** draw and label the starting 100% bar before finding any part. Use “starting,” “change” and “final” as three separate labels. Offer oral explanations or a calculator once the reference whole is chosen.
- **Extra challenge:** increase 12 by 150%. **Guide:** the increase is $1.5 \times 12 = 18$; the final amount is $12 + 18 = 30$, or 2.5×12. An increase larger than the original amount is possible.

⚠ Safety

Use fictional counts and lengths, not body measurements, personal income or comparisons between students.

Teacher notes

A final amount and a change are different quantities. “25% increase” means 125% remains relative to the original; “25% decrease” means 75% remains. Do not assume the same numerical change implies the same percentage when the starting whole changes. Percent change from an original amount of zero is undefined in this formula, not automatically 0% or 100%.

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Lesson 4: Two percentages, two reference wholes

Time: about 60 minutes (untested estimate; consider two sessions)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Solve successive and reverse percentage problems by identifying the reference whole at each step.

Materials

Paper, pencil and optional calculator. All prices, taxes and fees are invented for arithmetic, not statements of current policy or financial advice.

Steps

✎ 1. Opening — 5 minutes

O. Find 10% of 90. Explain what amount represents 100%.

✎ 2. Main activity: worked model — 15 minutes

A fictional item costs 80 currency units. Its price is discounted by 25%. A fictional 10% tax is then calculated **on the discounted price**. Find the total.

Write a label for the base at each stage:

Stage	Reference whole	Calculation
Discount	80	0.25 × 80 = 20
Price after discount	80	80 − 20 = 60
Tax	60	0.10 × 60 = 6
Total	60	60 + 6 = 66

The two multipliers give the same result: 80 × 0.75 = 60, then 60 × 1.10 = 66 currency units. Do not calculate 80 × (1 − 0.25 + 0.10) = 68: that incorrectly uses 80 as the base for both percentages. The problem expressly makes 60 the tax base.

Use a bar labelled 80 = 100%, with a quarter removed. Relabel the remaining 60 as 100% **for the tax step**, then add a tenth of that new whole. The quantity did not change when its percentage label changed; the reference did.

Explain a reverse problem: if a final price is 72 after a 20% markdown, the final amount is 80% of the unknown original x. Thus 0.80x = 72 and x = 72 ÷ 0.80 = 90. Check forward: 90

× 0.80 = 72. Adding 20% of 72 is not the inverse operation, because that uses the wrong base.

3. Guided work — 10 minutes

G. A length of 50 centimetres increases by 20%, then decreases by 20% of its **new** length. Find both intermediate and final lengths and the net percent change from the original. Before calculation, ask whether the two changes must cancel; return to that prediction after calculating.

✎ 4. Independent practice — 20 minutes

Use the stated bases, not assumptions about real shops. Write the expression before using a calculator. In these fictional problems, round monetary results to the nearest 0.01 currency unit only at the end; the supplied examples already terminate exactly.

- **P1.** A price of 120 currency units has a 15% discount, followed by a 10% tax on the discounted price. Find discounted price, tax and total.
- **P2.** A count of 200 grows by 10%, then by another 10% of the new count. Find the final count and overall percent increase from 200.
- **P3.** A final price is 72 currency units after a 20% markdown. Find the original and check it. Explain why 72 × 1.20 does not recover it.
- **P4.** Policy A adds a 10% tax on an 80-currency-unit item, then adds a flat fee of 5. Policy B taxes the combined item and fee, 80 + 5, at 10%. Find each total. Why are they different?

5. Discussion — 5 minutes

D. Does the worked example establish that every real tax is calculated after a discount? What information must a real question specify? Explain why P4 needs more than two percentage multipliers.

✓ 6. Closing check — 5 minutes

E. Start with 60. Decrease it by 10%, then increase the result by 10%. Find the intermediate amount, final amount and net percent change from 60.

Answer guide

- **O:** 0.10 × 90 = 9; the reference whole is 90.
- **G:** 50 × 1.20 = 60 centimetres; then 60 × 0.80 = 48 centimetres. The increase was 10 centimetres, but the decrease was 12. The net decrease is 2/50 = 0.04 = 4%, not zero.
- **P1:** Discounted price = 120 × 0.85 = 102. Tax = 102 × 0.10 = 10.20. Total = 112.20 currency units.
- **P2:** 200 × 1.10 = 220, then 220 × 1.10 = 242. Increase = 42; 42/200 = 0.21 = 21%. The second increase was 22, not 20.
- **P3:** 0.80 × original = 72, so original = 72/0.80 = 90 currency units. Check: 90 × 0.80 = 72. The proposed 72 × 1.20 = 86.40 adds 20% of 72, not the 18 removed from the original 90.

- **P4:** A: 80 × 1.10 + 5 = 93. B: (80 + 5) × 1.10 = 93.50 currency units. Policy B also taxes the fee, adding 0.50. Neither fictional policy is a claim about local law.
- **D:** “No: the problem must state the tax base and any fees. Real rules may differ. A flat fee is addition, so I cannot treat every operation as multiplication.” With the same two pure percentage factors and no intermediate rounding, their product commutes; this fact does not make all fee policies interchangeable.
- **E:** 60 × 0.90 = 54, then 54 × 1.10 = 59.40. The net decrease is 0.60/60 = 0.01 = 1%.

Vocabulary

- **Successive changes:** changes applied one after another, potentially with different reference wholes.
- **Reverse percent:** finding the original amount when the final amount and its percentage of the original are known.
- **Flat fee:** a fixed amount added, not a percentage of the amount bought.

Differentiation

- **Extra support:** provide two boxes headed “base for this step” and “new amount.” Complete the worked example with a bar before using a combined expression. Split successive changes and reverse percent into separate sessions if needed.
- **Extra challenge:** a final amount is 150 after a 25% increase. Find the original. **Guide:** 1.25x = 150, so x = 120; check 120 × 1.25 = 150.

⚠ Safety

Do not ask students to reveal family purchases, tax bills, debt or income. No real transactions are part of this lesson.

Teacher notes

Equal percentage increases and decreases generally do not cancel because the second base has changed. In reverse percent, undo the multiplier by division; do not merely switch “increase” and “decrease.” Require an explicitly stated base for every tax or fee example. Currency rounding policies vary in real life; the rule here is only an exercise assumption.

Previous lesson · Unit overview · Next lesson

Lesson 5: Choose the model, explain the result

Time: about 60 minutes (untested estimate; assessment may need a separate session)

Status: unreviewed AI draft; not classroom-tested. Author: GPT-6 Astra.

Goal

Choose and explain proportional and percentage models, including a stated simple-interest model, and check results using units and reference wholes.

Materials

Paper, optional grid or tactile grid, ruler and calculator. No financial accounts, internet access or personal financial information are required.

Steps

✎ 1. Opening — 5 minutes

O. A stated proportional relationship includes (2, 9). Find k and write its equation.

✎ 2. Main activity: simple-interest worked example — 15 minutes

In a fictional arithmetic model, the principal, or starting amount, is 200 currency units. Simple interest is paid at 3% per year for two years, with no fees or payments in between. **Each year's interest is calculated on the original 200**, not on a growing balance.

One year's interest = 200 × 0.03 = 6 currency units. Two years' interest = 6 × 2 = 12. Final amount = 200 + 12 = 212 currency units.

Write I = Prt, where I is interest, P is principal, r is the annual rate written as a decimal, and t is time in years. Here I = 200 × 0.03 × 2 = 12. State that the formula describes this simple-interest assumption; it is not a universal account rule or an investment recommendation.

With P = 200 and r = 0.03 fixed, interest is I = 6t, proportional to time. But the **total amount** is A = 200 + 6t. It is not proportional to time: at zero years, the total is already 200, not zero. This connects the percent calculation to lesson 2's fixed-fee distinction.

3. Guided work — 5 minutes

G. Use simple interest with principal 150 currency units, annual rate 4%, and time three years. Find yearly interest, total interest and final amount. Explain which amount is the reference whole each year.

4. Individual synthesis and assessment — about 25 minutes

This is not a speed test; allow more time or another session. Give the prompts before the guide. Accept equations, labelled diagrams or oral explanations. Ask students to show the rate or percent base, not only a final number.

- **A1. Fractional rate.** At a constant price, 5/8 metre of ribbon costs 15/16 currency unit. Find currency units per metre and the cost of 2 metres.
- **A2. Table and equation.** Hours x are 0, 2, 5; costs y in currency units are 0, 7, 17.5. Does the table fit a proportional model? Give k, the equation and the meaning of (1, 3.5).
- **A3. Straight but not proportional.** The rule is $y = 3 + 2x$, where x is hours and y is currency units. Its table includes (0, 3), (1, 5), (2, 7). Explain why the rule is not proportional even though its graph is straight.
- **A4. Graph interpretation.** Volume x in litres and cost y in currency units follow $y = 2x$ for continuous nonnegative volumes. Draw or describe a labelled graph including (0, 0), (1, 2) and (3, 6). Explain (3, 6) and how (0, 0) fits the model without dividing by zero.
- **A5. Change of base.** A count increases from 60 to 75, then later decreases from 75 to 60. Find the percent increase and percent decrease. Explain the difference.
- **A6. Successive percentages.** A fictional price of 150 currency units is discounted by 20%, then taxed at 5% of the discounted price. Find the final total, showing the tax base.
- **A7. Reverse percent.** An amount is 88 after a 10% increase. Find the original amount and check forward.
- **A8. Simple interest.** A fictional principal of 300 currency units receives 2% simple interest per year for four years, with no fees or intervening payments. Find the interest and final amount. Name the amount used as the percentage base each year.

5. Discussion — 5 minutes

D. Choose one assumption from today's models that would need checking in a real application. Explain what could change the answer if that assumption were false.

✓ 6. Closing check — 5 minutes

E. Explain why $I = 5t$ is proportional to time t but $A = 100 + 5t$ is not. Here t is years and both I and A are measured in currency units.

Worked answer guide

- **O:** $k = 9/2 = 4.5$; $y = 4.5x$. No units were specified in this abstract question.
- **G:** Yearly interest = 150 × 0.04 = 6 currency units; three years' interest = 18; final amount = 168. The original principal 150 is the reference whole every year.
- **A1:** (15/16) ÷ (5/8) = (15/16) × (8/5) = 3/2 currency units per metre. Two metres cost (3/2) × 2 = 3 currency units. Reverse units would answer a different question.
- **A2:** Yes: 7/2 = 17.5/5 = 3.5 currency units per hour; zero maps to zero. The fitted equation is $y = 3.5x$. (1, 3.5) means one hour costs 3.5 currency units under that model. The finite table fits; it does not alone prove unmeasured values.
- **A3:** At zero hours y = 3, not zero: the rule includes a fixed 3-currency-unit charge. Also 5/1 = 5 and 7/2 = 3.5 differ. The increase for each additional hour is constant, but the total cost per hour y/x is not.
- **A4:** Put volume in litres on the horizontal axis and cost in currency units on the vertical axis, with equally spaced numerical increments on each axis. The points lie on a straight line

through the origin; fractional inputs between them are permitted by the continuous-volume model. (3, 6) means 3 litres cost 6 currency units. At x = 0, y = 2 × 0 = 0; no 0/0 calculation is needed. The constant is 2 currency units per litre, read at (1, 2).
- **A5:** Increase = 15, based on 60: 15/60 = 0.25 = 25%. Decrease = 15, based on 75: 15/75 = 0.20 = 20%. The absolute changes match but their starting wholes differ.
- **A6:** Discounted price = 150 × 0.80 = 120. Tax base is 120, so tax = 120 × 0.05 = 6. Total = 126 currency units.
- **A7:** 1.10 × original = 88, so original = 88/1.10 = 80. Check: 80 × 1.10 = 88.
- **A8:** Yearly interest = 300 × 0.02 = 6 currency units. Four years' interest = 6 × 4 = 24. Final amount = 300 + 24 = 324. Every year's base is the original 300, not the balance after previous interest.
- **D:** Sample responses: “A fixed fee would change a pure cost-per-item model”; “Tax on the original rather than discounted price would change the tax”; “A rate observed during one interval might change later”; “Compound interest would use a growing balance rather than the original principal.” A specific assumption and consequence matter more than matching these words.
- **E:** I = 5t has constant I/t = 5 for positive t and gives I = 0 at t = 0. A = 100 + 5t starts at 100, so it is not of the form A = kt; its positive-time A/t is not constant. Its graph is nevertheless straight.

Feedback and next steps

Treat the competencies separately, not as a single speed or ability ranking:

- A1: if division is reversed, return to lesson 1's “one of what?” and units; if the setup is correct but arithmetic fails, support fraction division.
- A2–A4: distinguish a constant additional charge from a constant total ratio; revisit both the origin and a positive-input unit-rate point in lesson 2.
- A5: label the starting 100% whole separately for each direction, using lesson 3's bars.
- A6–A7: list each stage's base and check a reverse-percent answer forward, using lesson 4.
- A8: separate interest from final amount and simple from growing-balance rules.

After discussion, offer an analogous question rather than asking students to memorize this key. These prompts and feedback categories are not a validated assessment instrument; there is no established pass threshold.

Vocabulary

- **Principal:** the original amount used in the stated interest model.
- **Simple interest:** interest computed on the original principal for the stated time, rather than on accumulated interest.
- **Annual rate:** a rate per year; convert a percent to a decimal before using I = Prt.

Differentiation

- **Extra support:** provide one task per page, enlarged graphs or oral response options. Allow untimed work and a calculator after an expression is selected. Revisit the worked example before assessing simple interest separately.
- **Extra challenge, not required for assessment:** compare the worked example with annual compound growth at 3% for two years. **Guide:** 200 × 1.03 × 1.03 = 212.18,

compared with simple-interest total 212. The extra 0.18 is 3% of the first year's 6 interest. This illustrates a different assumption, not a full compound-interest course.

⚠ Safety

All money contexts are fictional. Do not request personal savings, borrowing, income or family circumstances. No purchases, account creation or financial recommendations are part of the activity.

Teacher notes

Separate “the interest is proportional to time” from “the total is proportional to time.” Keep rate periods and time units consistent: the displayed formula uses an annual rate with time in years. Classroom pacing and assessment use need local educator review. A second agent checking arithmetic does not make this a human-reviewed unit.

Previous lesson · Unit overview